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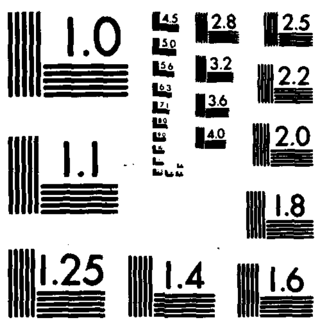

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TECHNICAL REPORT K-80-5

BASIC PILE GROUP BEHAVIOR

by

The CASE Task Group on Pile Foundations

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December 1980

Final Report

A report under the Computer-Aided Structural Engineering (CASE) Project

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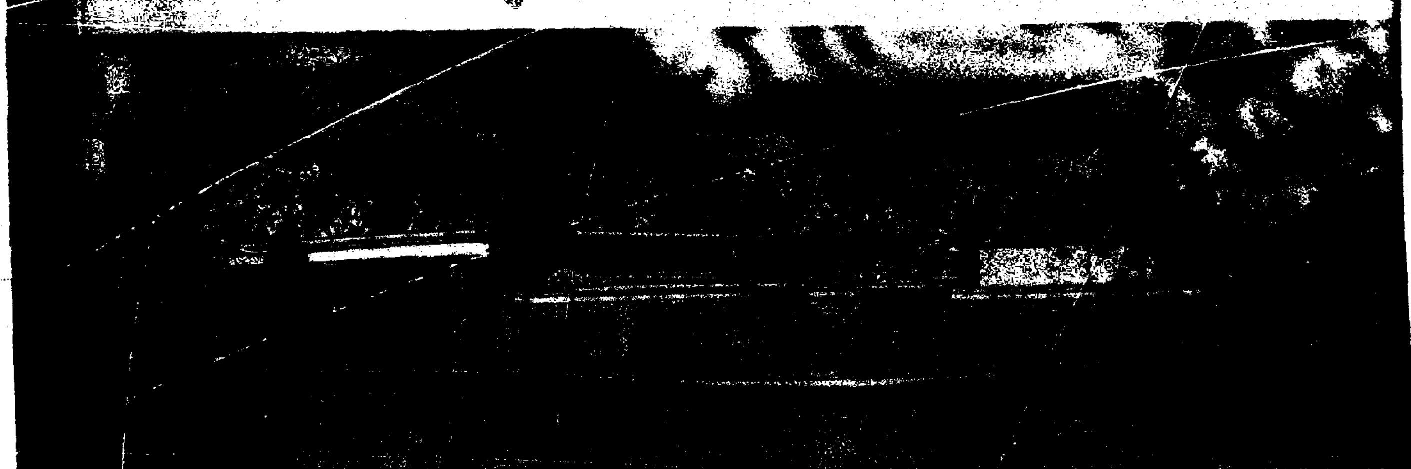

Prepared for Office, Chief of Engineers, U. S. Army
Washington, D. C. 20314

Monitored by Automatic Data Processing Center
U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report K-80-5	2. GOVT ACCESSION NO. AD-A093	3. RECIPIENT'S CATALOG NUMBER 696
4. TITLE (and Subtitle) 6 BASIC PILE GROUP BEHAVIOR		5. TYPE OF REPORT & PERIOD COVERED 9 Final report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) The CASE Task Group on Pile Foundations		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS 10 Joseph / Hartman		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 12 171
11. CONTROLLING OFFICE NAME AND ADDRESS Office, Chief of Engineers, U. S. Army Washington, D. C. 20314		12. REPORT DATE 11 December 1980
		13. NUMBER OF PAGES 166
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) U. S. Army Engineer Waterways Experiment Station Automatic Data Processing Center P. O. Box 631, Vicksburg, Miss. 39180		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Computer programs Computerized simulation Design criteria Pile foundation design Pile foundations		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes a computerized method for pile group design and analysis, as practiced by the Corps of Engineers, and proposes criteria for systematizing this method in a new computer program. Example problems are included. An overview of advanced methods of pile design and a brief discussion on selections of pile types, methods of installation, and allowable stresses are also presented. Estimation of soil moduli for laterally loaded piles and computation of pile stiffness coefficients for various soil conditions that are needed for the analysis are also discussed.		

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PREFACE

This report describes a computerized method for analysis and design of pile groups that is currently being used by several Corps of Engineers offices. Criteria for a new, comprehensive computer program for pile analysis and design are also discussed. The work was sponsored under funds provided to the U. S. Army Engineer Waterways Experiment Station (WES) by the Office, Chief of Engineers (OCE), under the Computer-Aided Structural Engineering (CASE) Project.

The report was compiled by the CASE Task Group on Pile Foundations. Members and others who directly contributed to the report were:

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CONTENTS

CONVERSION FACTORS, INCH-POUND TO METRIC (SI) UNITS OF MEASUREMENT

Inch-pound units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	2.54	centimetres
inch-pounds (force)	0.1129848	newton-metres
pounds (force) per inch	0.1751268	kilonewtons per metre
pounds (force) per square foot	47.880263	pascals
pounds (force) per square inch	6.894757	kilopascals
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
pounds (mass) per cubic inch	0.0276799	kilograms per cubic centimetre
tons (2000 lb mass)	907.18474	kilograms

I. SCOPE.

The purpose of this paper is to present one method for pile group design and analysis as practiced by the Corps of Engineers, and to propose criteria for systematizing this method in a computer program. This paper describes a computerized method of pile group analysis (including sample problems) and lists criteria for a new, more comprehensive program; it includes an overview of advanced methods of pile design, and briefly discusses selection of pile types, methods of installation, and allowable stresses

II. PILE DESIGN CONSIDERATIONS.

A. Economic. Pile foundations are a major cost in a structure. Pile foundations that provide the lowest first cost are of paramount importance. A cost comparison must be made of the relative cost of different type piles and cost of installation. Scheduling and availability may affect pile costs. Details affecting selection of pile type are presented in Appendix A.

B. Affect on Adjacent Structures. Proximity of adjacent structures may dictate the type of pile or installation used. Adverse effects of soil displacement or vibration caused by driving piles may compel the use of drilled caissons, nondisplacement piles, or jetting or predrilling of piles.

C. Difficulty in Installation. Hard strata, boulders, buried debris, and other obstructions may necessitate the use of piles durable enough to sustain driving stresses. Jetting, predrilling, or spudding may be required. Descriptions of installation methods are presented in Appendix B.

D. Environment. Corrosion in sea water will require consideration for protective coating, concrete jacketing, or cathodic protection if steel piling is used. The presence of marine-borers may negate the use of wood piling and the subsequent use of steel or concrete piling.

E. Displacements. Limitations on lateral or rotational movement will affect the type of pile used and the configuration of the pile group. Stiffer piles and the degree of fixity to the pile caps are considerations to limit displacements.

F. Foundation Materials. The capacity of the piling may be limited by failure of the foundation materials, evidenced by excessive settlement of piles under applied load. The capacity of the foundation materials is usually evaluated by static resistance formulas during design and verified by load tests prior to construction. Dynamic driving formulas are generally not a reliable basis for estimating pile capacities unless correlated with load tests and previous experience at similar, nearby sites. More reliable predictions of dynamic behavior during driving are based on complex computerized models of hammer-pile-soil- interaction using the ID wave equation.

G. Failure Modes.

1. Bearing capacity failure of the pile-soil system.

2. Excessive settlement due to compression and consolidation of the underlying soil.

3. Structural failure of the pile under service loads.

4. Bearing capacity failure caused by improper installation methods.

5. Structural failure resulting from detrimental pile installation. This may be due to unforseen subsoil conditions or to freeze, compaction, liquification, or heave of the soil. It could be caused by driving sequence, size of hammer, vibration, over or under driving, improper preexcavation methods, substitution of materials, improper workmanship, or

limitations of the Contractor's equipment or expertise. These conditions are described in detail in Chapter 2 of Reference 1.

H. Other Considerations. For more detailed discussion of the above considerations, and for others not mentioned above, see Reference 2.

III. BASIC PILE GROUP ANALYSIS.

This section presents the fundamentals of a basic method of pile group analysis which is currently available in various computer programs including LMVDPILE which is in WESLIB. Several hand analysis methods are shown with the sample problems in Appendix E. This computer method is capable of handling three-dimensional loading and pile geometry. It is valid for static analysis of a linear, elastic system. Interaction between pile and structure is limited to the extremes of a fully fixed or fully pinned connection. Interaction between the pile and soil is represented by a linear, elastic pile stiffness (applied load per unit deflection) at the top of the pile. The base of the structure is assumed to act as a rigid body pile cap connecting all piles; the cap flexibility is not considered.

A. Basic Analysis Method. The basic pile group analysis method represents each pile by its calculated stiffness coefficient, in the manner proposed by Saul (3). The stiffness coefficients of all piles are summed to determine a stiffness matrix for the total pile group. Displacements of the rigid pile cap are determined by multiplying the sets of applied loads by the inverse of the group stiffness matrix. Displacements of the rigid pile cap define deflections of individual pile heads which are then multiplied by the pile stiffness coefficients to determine the forces acting on each pile head. The key step in the method is in determining individual pile stiffness coefficients, at the pile head, based on known or assumed properties of pile

and soil. Since this is a three-dimensional analysis method, each pile head has six degrees of freedom (DOF), three translations and three rotations. A stiffness coefficient must be determined for each DOF and for all coupling effects (e.g. lateral deflection due to applied moment). The pile location and batter angle are also accounted for when individual pile stiffness coefficients are combined to form the total stiffness matrix for the pile group.

B. Pile-Structure Interaction. Piles are mathematically represented in the analysis by their axial, lateral and rotational stiffness, as springs resisting motion of the rigid cap. Such a system is shown in Figure 1.

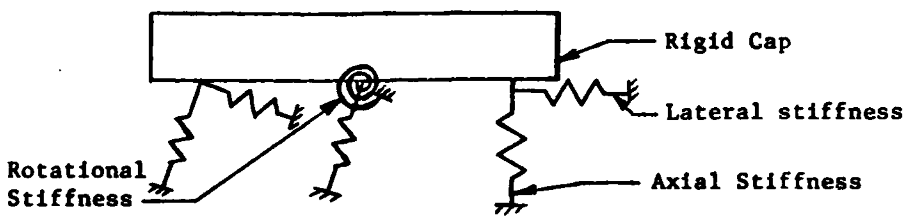


FIGURE 1

As mentioned above, consideration is given only to piles which are fully fixed or pinned to the pile cap. A pile embedded only a short distance into the cap may be assumed to transfer no moment at the pile head. Such a pile will resist only shear and axial loads. Well-embedded piles will resist shears, moments, and axial loads and will have coupling stiffness, referred to above. It is necessary to consider the fixity of the cap-pile joint to adequately determine pile stiffness. This should be done in conjunction with consideration of pile-soil interaction. Pile head fixity parameters have been derived by Dawkins (4). Once an analysis has determined the forces

acting on each pile, these forces may then be applied to the pile cap to determine its internal shears and moments. However, analysis of the pile cap is outside the scope of this section.

C. Pile-Soil Interaction. Interaction between the pile and soil is the most important consideration in determining pile stiffness. Therefore, it is necessary to have reliable information about soil properties. Soil properties can affect the axial, lateral, or torsional stiffness of the pile. The type of loading expected (static or cyclic) and the pile spacing should also be considered since cyclic loading or close spacing may both reduce individual pile stiffness.

1. Axial Stiffness. Axial load in a pile may be transferred to the soil by some combination of tip bearing and skin friction. For a pile transferring all load by tip bearing the axial stiffness is obviously AE/L, the stiffness of any axially loaded structural member. For a pile transferring all load by skin friction uniformly along its entire length, with no further tip movement, the axial stiffness is 2AE/L. Any other axial stiffness is possible for other types of piles, as shown in Figure 2.

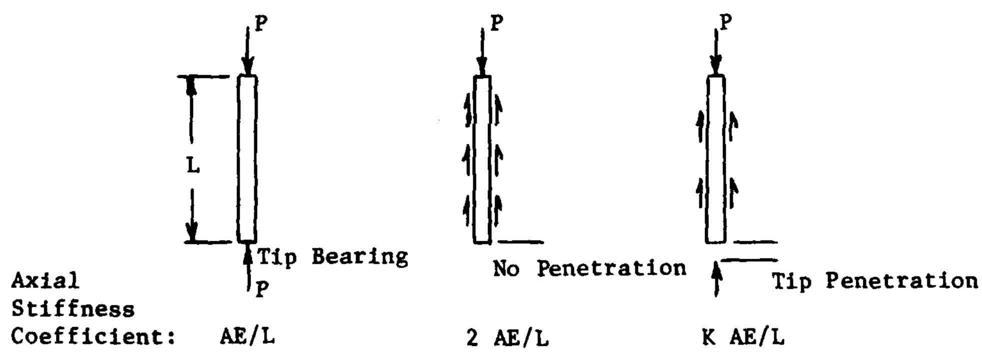


FIGURE 2

A further complication of pile axial stiffness involves consideration of tension piles. Generally, a pile in tension will be less stiff than the same pile in compression. Since only a single elastic stiffness coefficient may be specified for each pile, that stiffness must be based on whether the load is expected to be tension or compression.

2. Lateral Stiffness. Pile lateral stiffness refers to rotational stiffness and coupling effects, in addition to actual translational stiffness. The most important consideration is the resistance of the soil to translation of a pile. The degree of fixity between the cap and the pile must also be considered. The pile may be represented as a beam on elastic foundation, with the soil represented as a set of springs acting on the pile, as shown in Figure 3.

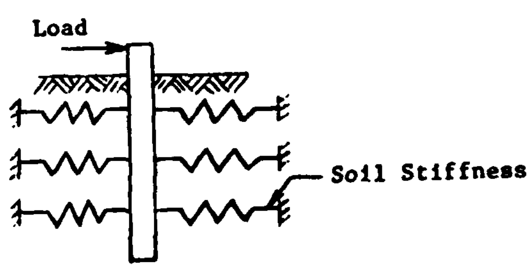


FIGURE 3

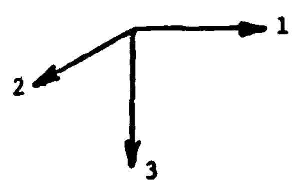
Though soil properties are often highly non-linear, an approximate linear lateral stiffness coefficient must be determined. Several analytical methods may be used to determine this stiffness. One method is to use any beam analysis computer program capable of representing the beam-spring system shown in Figure 3. The stiffness equals the force required to cause a unit displacement at the pile head. This method may be used to determine lateral, rotational and coupling stiffness coefficients. A method for determining appropriate values for the stiffness of the soil springs is included as Appendix D. Methods for determining pile stiffness coefficients are presented in detail in Appendix C.

3. Torsional Stiffness. For groups of piles, torsion on individual piles is usually unimportant and may be neglected by using zero torsional stiffness. Where torsion of individual piles is important, torsional stiffness may be determined in a manner similar to that described above for axial stiffness (5).

D. Analysis Details. As mentioned above, this analysis method has been systematized for use in computer programs. Several of these programs were identified during the Corps-Wide Conference on Computer Aided Design in Structural Engineering (6). Following are some of the detailed formulations used in these programs.

1. Coordinate System. The basic coordinate system is a right hand system, as shown in Figure 4. The three axes are labeled 1, 2, and 3, with the 3 axis being positive downward. This global coordinate system is used for specification of pile locations and orientations, applied forces and

moments on the pile cap, and for calculation of total pile group stiffness and resulting pile cap displacements.



Global Coordinate System

FIGURE 4

Each pile also has its own local coordinate system as shown in Figure 5. The axes are labeled 1, 2, and 3 and are located by specifying translations and rotations from the global coordinate system. The 3 axis is positive along the pile length, the 1 and 2 axes correspond to the pile principal axes, and the pile batter is in the local 1-3 plane.

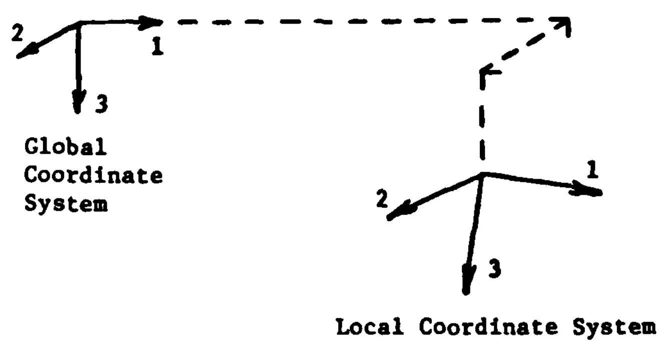


FIGURE 5

The local coordinate system is used for calculation of the stiffness coefficients, displacements, and forces of individual piles.

2. Pile Stiffness Matrix. For a pile with 6 degrees of freedom, individual pile stiffness coefficients are represented by a 6 x 6 matrix:

$$b = \begin{bmatrix} b_{11} & 0 & 0 & 0 & b_{15} & 0 \\ 0 & b_{22} & 0 & b_{24} & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & 0 \\ b_{51} & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{bmatrix}$$

Where 1, 2, and 3 refer to the pile coordinate system axes, and 4, 5, and 6 are rotations about those axes. Thus, b_{11} and b_{22} are lateral stiffnesses, b_{33} is axial stiffness, b_{44} and b_{55} are rotational stiffnesses, b_{66} is torsional stiffness, and b_{15}, b_{24}, b_{42}, and b_{51} are coupling stiffnesses.

b_{11} - is the force required to displace the pile head a unit distance along the local 1 axis

b_{22} - is the force required to displace the pile head a unit distance along the local 2 axis

b_{33} - is the force required to displace the pile head a unit distance along the local 3 axis

b_{44} - is the moment required to displace the pile head a unit rotation around the local 1 axis

b_{55} - is the moment required to displace the pile head a unit rotation around the local 2 axis

$*b_{15}$ - is the force along the local 1 axis caused by a unit rotation of the pile head around the local 2 axis

$*b_{24}$ - is the force along the local 2 axis caused by a unit rotation of the pile head around the local 1 axis

$*b_{51}$ - is the moment around the local 2 axis caused by a unit displacement of the pile head along the local 1 axis

$*b_{42}$ - is the moment around the local 1 axis caused by a unit displacement of the pile head along the local 2 axis

*Since the stiffness matrix must be symmetric $b_{15} = b_{51}$ and $b_{24} = b_{42}$. The sign of b_{24} and b_{42} must be negative.

Generally, each stiffness coefficient is influenced by the effects of pile-structure and pile-soil interaction. For example, b_{11} may be defined as:

$$b_{11} = C_1 C_2$$

Where C_1 is a constant depending on the pinned or fixed condition at the pile head and C_2 is a constant based on pile-soil interaction. Depending on the method used, these terms may be calculated separately and then multiplied to determine the pile stiffness coefficient, or the entire stiffness may be determined directly.

3. Analysis Method. The stiffness matrix of each pile is transformed from the local coordinate system to the global coordinate system. All pile stiffness matrices are then summed to form a 6 x 6 matrix

representing the stiffness of the entire pile group. Applied loads are defined as a set of three forces and three moments acting on the pile cap. To determine displacements of the pile cap the following equation must be solved:

$$[F] = [K]\ [U]$$

Where F is the applied load set, K is the pile group stiffness matrix, and U is the set of pile cap displacements. Once these displacements have been determined, the displacements at the head of each pile can be determined by a geometric transformation based on the location and orientation of that pile. The following equation must then be solved to determine forces acting on each pile head:

$$[f] = [b]\ [u]$$

Where f is the set of pile loads, b is the pile stiffness coefficients, and u is the set of pile head displacements. The above represents the basic analysis of a pile group. Further details are contained in the user's manuals for the various computer programs.

E. Limitations. Most of the limitations of this method of pile group analysis have been mentioned above, but will now be summarized. The method is valid for static analysis of a linear, elastic system. Applied loads must be equivalent static loads, non-linear soil properties must be represented by linear pile behavior. The other major limitation is that the pile cap is assumed to be rigid. Though this may be a valid assumption for a massive structure, such as a dam pier, it may result in gross errors in long, thin structures, such as a U-frame lock monolith.

F. Sample Analyses. Several sample problems are shown in Appendix E, solved by the above method and by conventional hand methods.

IV. COMPUTER PROGRAM CRITERIA.

A. General Requirements. Several related programs currently use the Saul method of pile group analysis, as described in the previous section. None of these programs has proven completely satisfactory to a wide range of users. The capabilities of several of these programs are shown in Table 1. The following paragraphs describe general criteria for a new program utilizing the Saul method. This program is intended to satisfy the widest possible range of users. Therefore, it must incorporate extensive capabilities and yet maintain a user oriented format. The necessary capabilities include the basic analysis and comparisons of calculated to allowable loads. The required user oriented features include convenient input formats and user control over program operations. A detailed criteria document will be published separately to fully describe the required capabilities of a new pile group analysis program.

TABLE 1 - Program Capabilities for Pile Group Analysis

	New Orleans 3D	LMVD PILE	St. Louis 3D
DOCUMENTATION			
User' Manual	X	X	X
Theorectical Manual		X	X
Example Problems			
INPUT			
Data File Input	X	X	X
Interactive Input		X	
Constant n_h	X	X	X
Constant E_s	X	X	

TABLE 1 - Program Capabilities for Pile Group Analysis (continued)

	New Orleans 3D	LMVD PILE	St. Louis 3D
INPUT (continued)			
Layered E_s			
Direct b_{ij} Input		X	X
Pile Coordinate Generation	X	X	
ANALYSIS			
Saul 2D			
Saul 3D	X	X	X
Vetters			
Tension Pile Interation			
Checks Calculated vs Allowable Loads	X	X	X
OUTPUT			
Input Echo	X	X	X
Pile Stiffness Coefficients	X	X	X
Pile Group Stiffness Matrix	X	X	X
Elastic Center		X	X
Structure Deflections	X	X	X
Pile Deflections		X	X
Pile Forces	X	X	X
Pile Force Components		X	X
Sum of Pile Force Components	X	X	X
Maximum Bending Moments For Pinned Piles	X		
Selective Output Items	X	X	X
GRAPHICS			
Pile Layout	X	X	X
Load Vectors vs Elastic Center		X	X
Pile Forces	X	X	X
Pile Load Factors		X	X

B. Program Operation. The user should have control over the specific operations to be performed by the program on any given run. To provide this control, the following capabilities are required.

1. Timesharing. The program should run in the timesharing mode since that is generally more convenient than batch execution.

2. Input Mode. The program should accept interactive input of data in response to program prompts. It should also accept input from a data

file. The interactive input is useful as a learning technique for new users, while data file input is a faster method for experienced users.

3. Output Routing. The program should be able to print output at the timesharing terminal or send selected data to a file.

4. Selective Output. The user should be able to select those items he wishes to see output. There should also be a capability to print output only for selected piles.

5. Tension Pile Iteration. The program should be able to iterate, at the user's option, to account for the extra flexibility of piles in tension.

C. Pile Layout Input. Since the pile layout description often constitutes the bulk of the input data, considerable effort should be given to simplifying this input.

1. Location. Pile locations should be specified, in feet, by X, Y, Z coordinates.

2. Pile Generation. Simple pile generation routines should be capable of easily describing common pile layouts such as equally spaced piles between end points or rectangular grids of piles.

3. Batter. Batter should be specified as a ratio of vertical to horizontal distance along the pile. The direction of the batter and the pile principal axis should be specified, in degrees, as the angle between the batter direction and the X-axis. Batters and angles should be specified in a simple manner, such as specifying a batter and then listing all piles which have that batter.

D. Pile Property Input.

1. Direct Stiffness Input. The user should be able to directly specify the coefficients of the individual pile local stiffness matrix.

2. Automatic Stiffness Calculation. The program should be able to compute the lateral stiffness coefficients automatically for the common cases of a constant or linearly varying modulus of horizontal subgrade reaction (K_h). The axial and torsional stiffness coefficients should be calculated as C_1AE/L and C_2JG/L, respectively. The length used in these calculations should be determined by the program based on the elevations of the pile head and pile tip, and considering the specified batter of the pile.

3. Pile Head Fixity. The lateral stiffness of a pile depends on the degree of fixity between the cap and the pile. The user should be able to specify this as fully fixed or fully pinned. The program should include this fixity when calculating the lateral stiffness coefficients.

E. Pile Allowable Loads. The program should check calculated pile loads against allowables specified by the user.

a. Axial Load. The allowable axial loads specified by the user may depend on soil capacity, on pile material capacity, or on pile buckling and should be compared directly to the calculated loads.

b. Bending and Axial Load. The user must specify allowable moments about both principal axes and an allowable axial load to be used in a combined stress equation.

c. Maximum Bending Moments. The program must calculate maximum bending moments about both axes for use in the combined stress equation. The maximum moments often occur at points other than the pile head.

d. Overstress Factors. The program should accept different allowable loads for different load cases to account for Group II loads.

F. Applied Loads. The user must define load cases as sets of three forces and three moments, referenced to the global coordinate system.

G. Output. The program should output, at the user's option, echoes of the pile locations, orientations, properties, and allowable loads; tables of calculated pile forces and combined stress factors for all piles, for selected piles, or for overstressed piles; and deflections of the pile cap and any specified points in space.

H. Pre-Processors and Post-Processors.

1. Individual Pile Behavior. A program should be available to calculate pile axial and lateral stiffnesses for any possible combination of pile and soil properties. This program should also be able to calculate and display the values of shear, moment, deflection and soil pressure along the entire length of any pile for specified pile head loads.

2. Graphical Displays. A program should be available to display the specified pile layout, including batters. It should also be able to display calculated pile forces and combined stress factors superimposed on a pile layout.

3. Pile Interference. A program should be available to check clearances between specified piles with different locations, batters and batter directions.

4. Base Slab Analysis. A post-processing program should be available to use pile forces, transformed to global coordinates, to help calculate shears and moments in portions of the pile cap.

V. ADVANCED METHODS OF PILE DESIGN.

A. PILEOPT Program. PILEOPT is a computer program intended to help determine the most economical pile layout possible for a given set of applied loads and within constraints specified by the user. It was developed by Dr. James L. Hill, under a contract with the Corps of Engineers. The program

uses the same analysis method described previously to determine pile forces for a given layout and applied loads. If the pile forces are less than the specified allowables, the program deletes some piles from the previous layout and reanalyzes. The program also attempts to choose the optimum batter for each pile group. The CASE Task Group on Pile Foundations will furnish a more detailed report on PILEOPT at some future date.

B. Flexible Base Analysis. The pile analysis method described above assumes that the pile cap, or structure base slab, is rigid in comparison to the stiffness of the piles. For many structures, such as U-frame lock monoliths, this is not a valid assumption, and the flexibility of the base slab should be considered. This requires use of large programs like SAP or STRUDL which can represent the stiffness of the structure and the piles. The pile element used in the rigid base method has been added to several versions of the SAP program and to a version of STRUDL. Flexible base analyses have already been performed for pile founded structures designed by the Corps of Engineers. A more detailed report on flexible base analysis will be furnished at some future date.

C. Non-Linear Analysis. One of the assumptions made in the rigid base analysis method is that a pile can be represented by a set of linear stiffnesses. The actual behavior of the pile-soil system may be highly non-linear. Some existing programs are capable of non-linear analysis of a structure which is supported by only a few piles. However, for large structures supported by many piles, non-linear analysis is not currently practical. A more detailed report on non-linear analysis will be furnished at some future date.

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APPENDIX A
PILE TYPES AND ALLOWABLE STRESSES

I. GENERAL. Representative values of allowable stresses for steel, concrete and timber piles are presented in this Appendix. This information is compiled from data published by technical societies, voluntary standards organizations, structural codes, and Corps of Engineers' guidance, and is intended only for general guidance.

II. TIMBER PILES. The trees most commonly used for piles in the United States are Douglas Fir, Southern Yellow Pine, Red Pine, and Oak. Timber piles are generally the most economical type for light to moderate loads. They are available in lengths from 30 to 60 ft.* Timber piles, however, are vulnerable to damage from hard driving and to deterioration caused by decay, insect attack, marine borer attack, and abrasive wear. Timber piles are commonly used for dolphins and fenders for the protection of wharves and piers because of their resilience and ease of replacement.

A. Allowable Design Stresses. Representative allowable stresses for pressure treated round timber piles for normal load duration are shown in TABLE A-1. These allowable stress values were derived by equations specified by ASTM D2899. "Standard Method for Establishing Design Stresses for Round Timber Piles". ASTM D2899 does not provide a method for establishing the allowable tensile stress parallel to the grain. However, an allowable tensile stress equal to the allowable bending stress may be used.

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* A table of factors for converting inch-pound units of measurement to metric (SI) units is presented on page 3.

Table A-1 Allowable Unit Stresses for Fully Supported, Pressure Treated, Round Timber Piles - Normal Load Duration (4) (7)

Species	Compression Parallel to Grain (psi) F_a(5) (6)	Bending (psi) F_b(6)	Horizontal Shear (psi)	Compression Perpendicular to Grain (psi)	Modulus of Elasticity (psi)
Pacific Coast (1) Douglas Fir	1050	2050	115	230	1,500,000
Southern Pine (1) (2)	1000	2000	110	250	1,500,000
Red Oak (3)	900	2050	135	350	1,280,000
Red Pine	750	1600	85	155	1,280,000

(1) The working stresses for compression parallel to grain in Douglas Fir and Southern Pine may be increased 0.2 percent for each foot of length from the top of the pile to the critical section. For compression parallel to grain, an increase of 2.5 psi per foot of length is recommended.

(2) Values are weighted averages for longleaf, slash, loblolly, and shortleaf.

(3) Values are weighted averages for Northern and Southern Red Oak.

(4) The working stresses in this table, excepted for modulus of elasticity, have been adjusted to compensate for strength reductions due to conditioning prior to treatment. There piles are air dried or kiln dried before pressure treatment, or where untreated piles are to be used, the above working

stresses shall be increased by dividing the tabulated values by the following factors:

Pacific Coast Douglas Fir, Red Oak, Red Pine:	0.90
Southern Yellow Pine:	0.85

(5) For allowable compressive stresses within the unsupported length of timber piles, see paragraph 1.B.

(6) The allowable stresses for compression parallel to the grain and bending, derived in accordance with ASTM D2899, are reduced by a safety factor of 1.2 in order to comply with the general intent of paragraph 13.1 of ASTM D2899.

(7) For hydraulic structures the values in this Table, except for modulus of elasticity, should be reduced by dividing by a factor of 1.2. This additional reduction recognizes the difference in loading effects between the ASTM normal load duration and the longer load duration typical of hydraulic structures, and the uncertainties regarding strength reduction due to conditioning processes prior to treatment.

B. Allowable Compressive Stresses for Unsupported Piles. The allowable compressive stress for cross sections within the unsupported length of timber piles may be determined by the formula:

$$F'_a = \frac{\pi^2 ES}{4.0\ (KL/r_A)^2}$$

where:

$$S = \frac{D_B}{D_A}$$

D_B = pile diameter at large end (point of connection to superstructure) (inches)

D_A = pile diameter at the location where pile is supported by soil (inches)

F'_a = allowable unit stress in compression parallel to the grain adjusted for KL/r ratio, when $F'_a < F_a$ (psi)

E = modules of elasticity of pile species (psi)

L = unsupported length of pile (inches)

r_A = radius of gyration of pile, taken at the location where the pile is supported by the soil (inches)

K = .7 for pinned-fixed end conditions

K = .5 for fixed-fixed end conditions

The above formula is applicable for a pile fixed below the ground surface and fixed (K = .5) or hinged (K = .7) at the pile cap. The formula has a safety factor equal to 4.0. If translation of the pile caps needs to be considered, a critical pile buckling load may be determined by methods outlined in reference (7) or by using the computer program discussed in Appendix C.

C. <u>Combined Axial Load and Bending</u>. For combined axial load and bending, stresses should be so proportioned that:

$$f_a/F_a + f_b/F_b \leq 1.0$$

where:

f_a = computed axial stress (psi)
f_b = computed bending stress (psi)
F_a = allowable axial stress (psi)
F_b = allowable bending stress (psi)

The above formula is applicable for:

$$\frac{KL}{D_A} \leq \sqrt{\frac{.15ES}{F_a}}$$

For $\frac{KL}{D_A} > \sqrt{\frac{.15ES}{F_a}}$, the combined axial load and bending

stress should be proportional that:

$$\frac{f_a}{F'_a} + \frac{f_b}{(1-f_a/f_b)\ F_b} \leq 1.0$$

where:

F'_a is as defined for unsupported piles

Since timber piles are tapered, the critical section or point of maximum stress may be at the tip for end bearing piles; or in the upper region where subject to bending, axial load and buckling; or at some point between for friction piles.

III. STEEL PILES.

Steel piles in general are available in long lengths; are able to withstand hard driving and penetrate dense strata; and can carry moderate to heavy loads. Embedded steel piles may be subject to deterioration; by rusting above and slightly below the ground line, especially in or near salt water; by corrosion if the surrounding foundation material is coal, alkaline soils, cinder fills or wastes from mines or manufacturing plants; or by local electrolytic action.

A. H Piles. H piles are nondisplacement piles which cause little disturbance to the surrounding soil during driving. H piles can carry loads up to 200 tons, however, the usual range is from 40 to 120 tons. Their length, although basically unlimited, typically ranges between 40 to 100 feet. H piles are easy to splice.

B. Open-End Pipe. Open-end pipe piles can also be considered nondisplacement piles, provided they are augered or otherwise cleaned out as they are driven. They can be installed in unlimited lengths and can carry moderate loads.

C. Closed-End Pipe. Closed-end pipe piles are displacement piles used when it is desirable to add volume to and compact the surrounding soil in order to increase the skin friction on the pile. This type of pile may cause heave of the surrounding piles and soil.

D. Allowable Design Stresses. Allowable design stresses for steel piles are shown in TABLE A-2. Allowable compressive stresses are given for both the lower and upper regions of the pile. Since the lower region of the pile is subject to damage during driving, the allowable compressive stress should be .28 Fy (10,000) psi. This value may be increased for pipe piles that are inspected for damage after driving. Bending and buckling effects are usually minimal in the lower region of the pile and need not be considered. The upper region of the pile may be subject to the effects of bending and buckling as well as axial load. Since this region (from about 15 feet below the ground surface to the pile cap) is not usually damaged during driving, a higher allowable compressive stress is permitted. The upper region of the pile is actually designed in the same manner as a steel column, with due consideration to lateral support conditions and combined stresses.

TABLE A-2 - Allowable Design Stresses for Steel Piles
Increase Allowable Stresses 33% (.67 Fy max) for
Temporary Loads. (3)

Code or Reference	Compression at Pile Tip (psi) F_a	Comp. Upper Region Subject to Combined Stresses (psi) F_a (4)	Tension (psi) F_t	Bending (psi) F_b(5)(6)
AISC		0.60 Fy (22,000)	0.60 Fy (22,000)	0.60 Fy (22,000)
AASHTO		0.472 Fy (17,000)	0.55 Fy (20,000)	0.55 Fy (20,000)
Draft Pile (2) EM	0.28 Fy (10,000)	0.47 Fy (17,000)		0.60 Fy (22,000)
Recommended (1) for Hydr. Structures	0.28 Fy (10,000)	0.47 Fy (17,000)	0.50 Fy (18,000)	0.50 Fy (18,000)

(1) The recommended allowable stresses for hydraulic structures are 5/6 of AISC values. F_a for the upper region is based on an avereage safety factor rather than the variable safety factor specified by AISC.

(2) Note inconsistencies in Draft Pile EM values for head compression and bending.

(3) Values given in parenthesis are for A 36 steel.

(4) For allowable compressive stresses within the unsuppported length of steel piles, see paragraph 2E.

(5) For combined axial load and bending, see paragraph 2.F.

(6) The allowable bending stress values given assume the compression flange is adequately supported. For other conditions refer to the allowable bending stress formulas give in EM 1110-1-2101.

E. Unsupported Piles. The allowable compressive stress for an unsupported steel pile, where $Cc > \frac{KL}{r}$ may be determined by the formula:

$$F_a = \frac{F_y}{F.S.}\left[\frac{1 - (KL/r)^2 F_y}{4\pi^2 E}\right]$$

or when $Cc < \frac{KL}{r}$ by the formula:

$$F_a = \frac{\pi^2 E}{F.S. (KL/r)^2}$$

where:

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

and where:

F_a	=	allowable axial compressive stress (psi)
F_y	=	specified minimum yield stress (psi)
E	=	modulus of elasticity (29,000,000 psi)
L	=	actual unbraced length of pile from the pile cap to the point of fixity below the ground surface (inches)
K	=	effective length factor as defined by AISC
r	=	least radius of gyration (inches)

and where:

F.S.
= Factor of Safety
= Varies from 1.67 to 1.92 for AISC
= 2.12 for AASHTO
= 2.15 for recommended value for hydraulic structures

F. Combined Axial Load and Bending. Steel piles subject to axial load and bending shall be proportioned to satisfy the following requirements:

$$\frac{f_a}{F_a} + \frac{C_{mx} \quad f_{bx}}{(1-f_a/F'_{ex})F_{bx}} + \frac{C_{my} \quad f_{by}}{(1-f_a/F'_{ey})F_{by}} \leq 1.0$$

and:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \text{ (when } \frac{f_a}{F_a} \leq 1.5\text{)}$$

where:

$$F'_e = \frac{\pi^2 E}{F.S.\ (K_b\ L_b/r_b)^2}$$

and:

f_a	=	computed axial stress (psi)
f_{bx} or f_{by}	=	computed compressive bending stress about the x axis and y axis, respectively (psi)
F_a	=	allowable axial stress (psi)
F_{bx} or F_{by}	=	allowable compressive bending stressabout the x and y axis, respectively (psi)
E	=	modulus of elasticity (29,000,000 psi)
L_b	=	actual unbraced length of pile in the plane of bending (inches)
K_b	=	effective length factor as defined by AISC in the plane of bending (inches)
r_b	=	radius of gyration in the plane of bending (inches)
C_{mx} or C_{my}	=	coefficient about x and y axes, respectively, as defined by AISC
F.S.	=	Factor of Safety (see paragraph E)

G. <u>Splices</u>. Splices should be designed to develop the full strength of the pile in compression, tension, and flexure.

IV. CONCRETE FILLED STEEL PILES.

A. <u>Open- and Closed-End Pipe</u>. Pipe piles, both open- and closed-end, can be filled with concrete to increase their structural load-carrying capacity. Loads up to 300 tons can be carried with this type of pile.

B. <u>Drilled in Caissons</u>. Drilled in caissons are typically open-end pipe piles of 24-inch or 30-inch diameter, drilled into rock. They can carry loads up to 300 tons. If an H Pile core section is also used, the load carrying capacity can be increased considerably.

C. <u>Allowable Design Stresses</u>. Allowable design stresses for concrete filled steel piles should follow steel and concrete allowable stresses specified in paragraphs III and V of this Appendix.

D. Unsupported Piles and Combined Stresses. For these conditions, follow the provisions for concrete piles.

V. CONCRETE PILES.

A. Precast Concrete Piles. This general classification covers both conventionally reinforced concrete piles and prestressed concrete piles. Both types can be formed by casting, spinning, or extrusion methods, and are made in various cross section shapes such as square, octagonal, and round. Precast concrete piles must be designed and manufactured to withstand handling and driving stresses in addition to service loads.

1. Conventionally reinforced concrete piles are constructed of reinforced concrete with internal reinforcement consisting of a cage made up of several longitudinal bars and lateral ties of hoops or spirals.

2. Prestressed concrete piles are constructed using steel rods, strands, or wires under tension to replace the longitudinal steel used in the construction of conventionally reinforced concrete piles. The prestressing steel is enclosed in a conventional spiral. Such piles can usually be made lighter and longer than normally reinforced concrete piles for the same rigidity and bending strength. Other advantages of prestressed piles are:

a. Durability

b. Crack free during handling and driving

c. High load-carrying capacity

d. High moment capacity

e. Excellent combined load-moment capacity

f. Ability to take uplift (tension)

g. Ease of handling, transporting, and driving

h. Economy

i. Ability to take hard driving and to penetrate hard strata

j. High column strength

k. Readily spliced and connected

B. Cast-in-Place Concrete Piles. In general, cast-in- place concrete piles are installed by placing concrete in a preformed hole in the ground to the required depth. Depending on foundation conditions, the hole is usually lined with a steel casing which is left in place or may be pulled as concrete is placed. Since the concrete is not subjected to driving stresses, only the stress from service loads need be considered in the design. Basic types include the following: Cased driven shell, drilled-in-caisson, dropped-in-shell, uncased, compacted, auger grouted injected, cast-in-drilled-hole, and composite concrete piles. Detailed descriptions of each of these types are covered in Chapter 1 of Reference 1.

C. Allowable Design Stresses. The allowable design stresses determined in accordance with the recommended formulas in this section relate to the structural capacity of the pile with an applied factor of safety. The design stresses reflect a minimum safety factor of 2.2 (based on strength design) and include an accidental eccentricity factor of 5 percent. Allowable design stresses for concrete piles are shown in TABLE A4. For bond and shear allowables, see the provisions of ACI 318-77.

TABLE A-3 - Allowable Design Stresses for Concrete Piles

Allowable Stresses* (psi)	Permanent Loads	Hydraulic Structures
Concrete		
Compression		
Confined**	.40 f'c	.35 f'c
Unconfined	.33 f'c	.33 f'c
Tension		
Plain and Reinforced	0	0
Prestressed	$3\sqrt{f'c}$ (250 max)	$3\sqrt{f'c}$ (250 max)
Bending Compression		
All Types	.45 f'c	.35 f'c
Bending Tension		
Plain	0	0
Reinforced	0	0
Prestressed	$3\sqrt{f'c}$ (250 max)	$3\sqrt{f'c}$ (250 max)
Reinforcing Steel		
Grade 40, 50	20,000	20,000
Grade 60	24,000	20,000

*Reduce allowable stresses 10% for trestle piles and for piles supporting piers, docks, and other marine structures.

**Provided the steel shell confining the concrete is not greater than seventeen inches in diameter; is fourteen gage (U.S. Standard) or thicker; is seamless or has spirally welded seams; has a yield strength of 30,000 psi or greater; is not exposed to a detrimental corrosive environment; and is not designed to carry a portion of the pile working load.

D. Combined Axial Load and Bending. For combined axial load and bending, the concrete stresses should be so proportioned that:

1. Axial compression and bending:

For all piles: $\frac{fa}{Fa} + \frac{fb}{Fb} \leq 1.0$

For prestressed piles: $fa + fb + fpc \leq 0.45\ f'c$ (.35 f'c for hydraulic structures, compression)
$fa - fb + fpc \geq 0$ (tension)

2. Axial tension and bending:

For prestressed piles: $- fa - fb + fpc \geq 0$

where:

fa = actual axial stress
fb = actual bending stress
fpc = effective prestress after losses
Fa = allowable axial stress
Fb = allowable bending stress

3. When the pile is designed for combined axial load and bending, the working stress design should be checked using strength design methods to insure that the required minimum factor of safety is achieved in accordance with ACI 318-77.

E. Allowable Design Loads.

1. Laterally Supported Piles. The allowable compressive design loads on laterally supported concrete piles may be determined by using Table A-4.

2. Unsupported Piles. Where the pile extends above the ground or where scour is expected, the allowable load must be reduced. For l/r ratios up to 120, the allowable load for the unsupported pile length may be determined by applying a reduction factor R, to the allowable load for a fully supported pile, where $R = 1.23 - 0.008\ (l/r) = 1.0$. If l/r exceeds 120, the pile should be investigated for elastic stability. The effective pile length (l) is determined by multiplying the structural pile length (L) by the appropriate value of the coefficient K listed below:

VALUES FOR K FOR VARIOUS HEAD AND END CONDITIONS

Head Condition	End Conditions		
	Both fixed	One fixed	Both hinged
Non-translating	0.6	0.8	1.0

F. Other Considerations.

The pile foundation design should include other considerations to ensure that piles are installed satisfactorily. Some of these considerations are as follows:

1. Pile Dimensions. It is recommended that the minimum dimension be 10 inches.

2. Pile Shells. Pile shells or casing should be of adequate stength and thickness to withstand the driving stresses and maintain the cross section of the driven pile.

APPENDIX B
PILE INSTALLATION METHODS

I. DRIVING BY IMPACT METHODS.

Most piles are installed by driving with impact hammers. These hammers are usually powered by steam, air, or diesel. The pile driving equipment used should be adequate to satisfactorily install the pile to the penetration or resistance required without damage to the pile. Hammer types can be classified as gravity or drop hammers, single acting hammers, double acting hammers, or diesel hammers. Gravity or drop hammers are seldom used. They consist of a weight lifted by cable to a specified height. The weight is released and the energy, supplied by gravity, drives the pile. The single acting hammer operates in the same fashion, only the weight is raised by steam or air power. The steam or air power permits the weight to be raised and released much more rapidly than by drop hammer. Double acting steam hammers employ steam or air power to raise the hammer and to power the hammer on the downward stroke. Diesel pile hammers get their energy from the compression blow of a falling weight and the reaction to controlled instantaneous burning and expansion of fuel, which raises the hammer for the next stroke. In general, the more driving energy delivered to the pile, without damaging the pile, the better.

II. PRE-EXCAVATION METHODS.

Pre-excavation methods such as jetting, preboring, augering, or spudding are used when piling must be driven through dense or hard materials to bearing at greater depth or when it is necessary to remove an equivalent amount of non-compressible soil before installing displacement piles such as closed-end pipe, concrete, or timber. Pre-excavation methods will also

minimize or eliminate the vibration caused by driving which may damage adjacent structures. Pre-excavation methods should be used with care in order to insure the desired capacity of the piles being installed and the capacity of the piles already in place; and to insure the safety of nearby existing structures.

A. Jetting. Jetting is accomplished by pumping water through pipes attached to the side of the pile as it is driven. This method is used to install piles through cohesionless soil materials to greater depths. The flow of water reduces skin friction along the sides of the pile. Air jetting is also used. The pile is usually jetted to within a few feet from the final elevation and then driven. Since jetting reduces skin friction, it should be used with caution, especially for tension piles.

B. Predrilling or Augering. Predrilling is used to produce a hole into which a driven pile may be installed. The hole may be used to penetrate difficult materials or to provide accurate location and alinement of the pile.

C. Spudding. Spudding is accomplished by driving a heavy pipe section, mandrel or H pile section to provide a hole through difficult or hard foundation materials. The spud is pulled and the pile is inserted in the hole and driven to the required resistance.

III. VIBRATORS.

A. Low Frequency Vibrators. Low frequency vibrators deliver their energy by lifting the pile and driving it downward on each cycle. These operate at frequencies of 5 to 35 cycles per second. The vibration tends to reduce the frictional grip of the soil on the pile and the pile itself is used to impact the soil and overcome point resistance. This method has found only limited use in driving displacement piles. Use in the installation of nondisplacement piles, however, is increasing.

B. High-Frequency Vibrators. High frequency vibrators operate at the natural frequency of the pile. The pile itself imparts energy to displace the soil in front of the pile tip. High frequency vibrators operate between 40 and 140 cycles per second. Displacement piles over 100 feet long have been installed using this method.

IV. CAST-IN-PLACE PILES.

This method consists of forming a hole in the soil and filling it with concrete. Cast-in-place piles may be cased or uncased. Casings (shells) may be driven with or without a mandrel. The casings are driven to the desired resistance and filled with concrete or the casing may be slowly withdrawn as the concrete is poured into the hole.

APPENDIX C

PILE STIFFNESS COEFFICIENTS

I. GENERAL.

The ability of a pile foundation to resist applied loads depends on the complex interaction of the pile with the surrounding soil. The numerous factors which affect the response of a pile foundation must be reduced to a mathematical representation so that a reasonably accurate analytical evaluation can be performed. The most common method of accomplishing this representation is to replace the soil and pile with springs at the pile-structure interface. Once the various properties of the soil-pile foundation are represented by equivalent spring constants, it is relatively easy to determine the pile forces by use of any of several computer programs currently available. The difficulty arises in establishing the equivalent soil-pile springs with a reasonable degree of accuracy. Two approaches are generally used:

- Pile load test values: determined by actual full scale pile load tests at the construction site or a nearby site with similar soil conditions.

- Semi-Empirical: determined in two ways, by formulae or by computer solution. If the soil modulus* can be assumed to be constant or to vary linearly with depth, the equivalent springs can be determined directly by formulae shown in Table C-2. For more complex soil systems, a computer solution can be used to account for multi-layered soils, non-linear variation of soil modulus, and inelastic soil behavior by analyzing a single, isolated pile using known soil parameters for the site.

*NOTE: In this Appendix E_s refers to the horizontal soil modulus.

The equivalent springs or stiffness coefficients, determined by the methods described above are the "b" terms of the pile stiffness matrix as described in paragraph III D2 of the text. Generally, these terms can be defined as:

$$b_{ij} = C_1 C_2$$

where C_1 is a constant which depends on the fixity of the pile head to the structure. For most applications, a fixity condition of fully pinned or fully fixed is assumed. C_2 is a constant based on the pile-soil interaction and is determined by one of the methods mentioned above.

Values for the fixity constant C_1, for soils with a constant or a linearly varying soil modulus, E_s, are shown in Table C-1. A theoretical derivation of these values can be found in references 16 and 17.

TABLE C-1

Pile Fixity Constants, For Soils With
Constant or Linear Variation of Soil Modulus

Pile Stiff. Coeff.	Values of C_1			
	Const. Es		Linear Var. Es	
	Pile Head Fixity		Pile Head Fixity	
	100%	0%	100%	0%
b_{11}	2.0	1.0	1.075	0.411
b_{22}	2.0	1.0	1.075	0.411
b_{44}	1.0	0	1.5	0
b_{55}	1.0	0	1.5	0
b_{15}	1.0	0	1.0	0
b_{24}	1.0	0	1.0	0
b_{42}	1.0	0	1.0	0
b_{51}	1.0	0	1.0	0

The pile stiffness coefficients b_{33} and b_{66} representing the axial and torsional stiffness, respectively, of the pile are not shown in Table C-1. These two coefficients are assumed to be not affected by the pile head fixity and, therefore, are not shown. For additional discussion of the axial and torsional pile stiffness coefficients, see sections III and IV, respectively, of this appendix.

The pile stiffness coefficients can be affected by many factors other than the pile fixity constants (C_1) and the pile-soil stiffness constants (C_2). The major factors are mentioned here but a detailed discussion is beyond the scope of this appendix. The following items could influence the pile stiffness coefficients:

- Group effect: close spacing of piles in a large group can reduce the lateral capacity for the group.

- Position in group: a pile may exhibit different stiffness depending on its location in the group.

- Cyclic loading: repeated application of static loads on a pile can cause greater deflections of the pile than the application of a sustained static load of equal magnitude.

- Vibratory or dynamic loading: statically loaded piles subjected to vibrations or dynamic loads may deflect significantly more than with the static load only.

- Driving a pile into closely spaced group: when piles are driven in an area that already contains closely spaced pile, the soil density within the pile group can be affected.

- Sheet pile cutoff: sheet pile used to inclose pile groups may change the distribution of stress in the soil.

- Water table and seepage: the groundwater table and seepage can influence the lateral soil modulus.

- Pile length: short rigid piles act differently than long flexible piles. This report assumes piles are long enough to act in a flexural mode (non-dimensional length L/T is greater than 5, as defined by Reese (17)).

- Stiffness of pile cap: the flexibility of the pile cap will influence the distribution of load to the piles.

For additional discussion of the factors mentioned above, see reference 18.

The remainder of this appendix will deal with determination of the pile stiffness constants (C_2) without regard to the items briefly referred to above.

II. LATERAL STIFFNESS.

A. General. For structures which experience lateral loads of any significance, the correct representation of the lateral stiffness of the foundation in the anlysis is critical. This representation must include the resistance of the pile to lateral translation and rotation and the coupling effects. These stiffnesses are inserted in the pile stiffness matrix as the terms b_{11}, b_{22}, b_{44}, b_{55}, b_{15}, b_{24}, b_{42}, and b_{51}. These terms can be determined either by pile load tests or by semi-empirical methods.

B. Pile Load Tests. The pile stiffness coefficients can be determined by full scale pile load tests at the construction site or a nearby site with similar soil conditions. However, pile load tests may not be practical for design for several reasons:

1. The tests are usually very costly and time consuming and may not be economically feasible for small to medium size jobs.

2. Normally, pile load tests at the construction site are not conducted until construction is well underway. Since pile analysis and design must be accomplished well in advance of construction, data obtained from load tests could not be used for design but only for verification or modification of the pile design.

3. With restricted site areas, the pile load tests can be in the way of other construction and, in some instances, actually delay construction.

C. Semi-Empirical Methods. These methods can be categorized as analytical (using formulae) or as numerical (using a computer solution).

1. Analytical Method. If a soil system can reasonably be assumed to have a soil modulus that varies linearly with depth or that is constant, then the lateral stiffness constants can be calculated using prescribed values or ranges of values of the soil modulus. Shown in Table C-2 are formulae for calculating the lateral stiffness terms (b_{11} and b_{22}), the rotational stiffness terms (b_{44} and b_{55}), and the coupling stiffness terms (b_{15}, b_{24}, b_{42}, b_{51}).

These terms are defined as:

b_{11} is the force required to displace the pile head a unit distance along the local 1 axis

b_{22} is the force required to displace the pile head a unit distance along the local 2 axis

b_{33} is the force required to displace the pile head a unit distance along the local 3 axis

b_{44} is the moment required to displace the pile head a unit rotation along the local 1 axis

b_{55} is the moment required to displace the pile head a unit rotation along the local 2 axis

*b_{15} is the force along the local 1 axis caused by a unit rotation of the pile head around the local 2 axis

*b_{24} is the force along the local 2 axis caused by a unit rotation of the pile head around the local 1 axis

*b_{51} is the moment around the local 2 axis caused by a unit displacement of the pile head along the local 1 axis

*b_{42} is the moment around the local 1 axis caused by a unit displacement of the pile head along the local 2 axis

*Since the stiffness matrix must be symmetric $b_{15} = b_{51}$ and $b_{24} = b_{42}$. The sign of b_{24} and b_{42} must be negative.

TABLE C-2
PILE STIFFNESS COEFFICIENTS

Pile Stiff. Coeff.	Constant Es	Lin. Var. Es
b_{11}	$C_1 \frac{E_s}{2\beta_2}$	$C_1 \frac{EI_2}{T_2^3}$
b_{22}	$C_1 \frac{E_s}{2\beta_1}$	$C_1 \frac{EI_1}{T_1^3}$
b_{44}	$C_1 \frac{E_s}{2\beta_1^3}$	$C_1 \frac{EI_1}{T_1}$
b_{55}	$C_1 \frac{E_s}{2\beta_2^3}$	$C_1 \frac{EI_2}{T_2}$
b_{15}	$C_1 \frac{E_s}{2\beta_2^2}$	$C_1 \frac{EI_2}{T_2^2}$

TABLE C-2 (Continued)
PILE STIFFNESS COEFFICIENTS

Pile Stiff. Coeff.	Constant Es	Lin. Var. Es
b_{24}	$-C_1 \frac{E_s}{2\beta_1}$	$-C_1 \frac{EI_1}{T_1^2}$
b_{42}	$-C_1 \frac{E_s}{2\beta_1}$	$-C_1 \frac{EI_1}{T_1^2}$
b_{51}	$C_1 \frac{E_s}{2\beta_2^2}$	$C_1 \frac{EI_2}{T_2^2}$

where:

C_1 is the pile fixity constant as shown in Table C-1 and varies from one "b" term to another.

$T_1 = \sqrt[5]{\frac{EI_1}{n_h}}$ (in.) ; $T_2 = \sqrt[5]{\frac{EI_2}{n_h}}$ (in.)

n_h is the constant of horizontal subgrade reaction or the change in the soil modulus with depth (lb/in^3).

E_s is the horizontal soil modulus (lb/in^2).

$\beta_1 = \sqrt[4]{\frac{E_s}{4\ EI_1}}$ (in.) ; $\beta_2 = \sqrt[4]{\frac{E_s}{4\ EI_2}}$ (in.)

E is the modulus of elastically of pile (lb/in^2).

I is the moment of inertia of pile (in^4).

Subscripts 1 and 2 for I, T, and β refer to the local pile axes. See Figure C-1.

LOCAL PILE AXIS

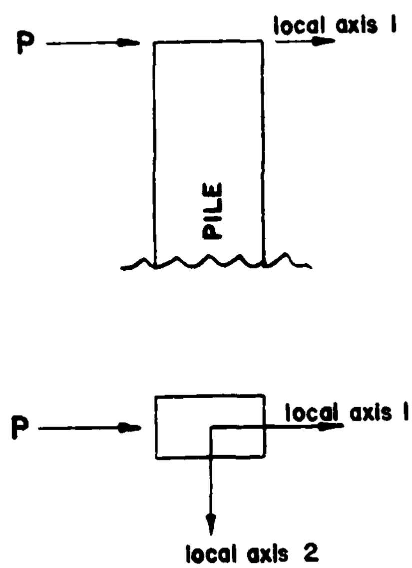


FIGURE C-1

The constant of horizontal subgrade reaction (n_h) or the horizontal soil modulus (E_s) can be obtained using methods shown in Appendix D. These methods are based on work by Terzaghi (19), Broms, and others and include corrections for pile group effect and for cyclic loading. These methods of soil modulus are satisfactory if the variation of the soil modulus with depth can be reasonably approximated as constant or linear. Many foundation strata fall in this category (12, 17) and can be conservatively represented by using a "bracket" approach to the pile design. This means the pile foundation is analyzed with weak pile stiffness coefficients and strong pile stiffness

coefficients, where "weak" and "strong" refer to the range of soil modulus that could reasonably be expected for a particular soil. In cases where the simplified assumptions are not valid, computer solutions, are needed.

2. Numerical Solution by Computer. Most analytical methods are based on a pile-soil system similar to a beam on elastic foundation. These methods assume that the soil can be represented by a series of closely spaced, independent springs. The pile-soil relationship can be expressed by a 4th order differential equation which can be solved for specific cases by making certain assumptions. There are several computer programs available which can be used to determine the pile stiffness coefficients for a single pile. Some of the most useful programs are discussed in the following paragraphs.

a. "Pile Head Stiffness Matrices." This program was written by Dr. William Dawkins for WES. This program is intended to be used to analyze a single pile to determine the stiffness coefficients for input to a general pile foundation analysis program. The procedure used is a one-dimensional analysis of a beam on an elastic foundation where the soil is represented as discrete springs. The soil springs are calculated by the program based on a variation of the lateral soil modulus of $E_s = a + bz^n$.

where: E_s = lateral soil modulus
a, b = constants
z = depth below ground surface

The values of "a", "b", "z", and "n" are input by the user. Any degree of fixity for the pile head to the pile cap can be considered with this program. Output consists of the actual pile stiffness coefficients ("b" terms) and may be used directly as input to a general pile foundation analysis program. Disadvantages of this program are that the user must know

the variation of soil modulus with depth and that the soil springs used are linearly elastic. Also the current version of the program does not contain provisions for variation of the pile stiffness with depth or for the application of axial loads. For information on this program see reference 4.

b. "Analysis of Laterally Loaded Piles by Computer". Several programs are available which can be used to determine the pile stiffness coefficients if the values of the soil springs can be determined by other means. The values for the soil springs are input to the program and the springs are treated as completely elastic or elastic-plastic, depending on the program's capabilities. Any variation of the soil modulus with depth can be represented by inputting the proper values for the discrete soil springs. Axial loads and variation of pile stiffness with depth can generally be accounted for in these programs. Output usually consists of values for the deflection, moment, and soil reaction for specified increments along the pile model. The pile stiffness coefficients ("b" terms) can be obtained by applying displacements and rotations to the pile model and then using the output of forces, moments, and displacements to determine the appropriate "b" terms. For example, to determine "b_{11}", which is the force required to displace the pile head a unit distance along axis 1, apply a force at the pile head along axis 1. Then "b_{11}" is equal to the displacement of the pile head divided by the applied force. The other "b" terms can be calculated in similar manner.

It should be noted that when the soil response to applied loads is non-linear (as it is assumed to be in this discussion) the pile head moments and displacements will vary non-linearly with the forces applied to the pile. For example, if the applied lateral force along the pile axis 1 is

increased linearly, the pile displacements will increase non-linearly and therefore "b_{11}" will not be a constant but will vary. In order to account for this non-linearity, the designer should determine the sensitivity of the particular foundation to variations in applied pile loads. This can be accomplished by comparing results from the application of small and large loads in the single pile analysis. If the foundation is determined to be sensitive to the load variations then the designer could account for this in the analysis by using a bracket approach for the "b" terms or by determining one set of "b" terms which reflect expected applied pile loads.

One of the more useful single pile analysis computer programs is "Analysis of Laterally Loaded Piles by Computer," written by Dr. Lymon Reese. For this program, soil properties are defined by a set of curves which give soil reaction as a function of pile deflection. The lateral resistance of the soil is represented by non-linear, discrete springs called p-y curves. These curves have been constructed for various soil conditions based on pile tests, theories for the behavior of soil under stress, and failure mechanisms for pile-soil systems. The program performs an iterative solution which consists of finding a set of elastic deflections of the pile which simultaneously satisfy the specified non-linear, resistance-deformation relations (p-y curves) of the soil and the elastic bending properties of the pile. This program can account for changes in pile types with depth; applied axial loads; a layered, non-linear, soil system; and any degree of fixity of the pile head to the structure. The p-y curves are a necessary input to this program.

Another program, "Generation of Soil Resistance versus Pile Movement Curves," is available which will generate the required family of p-y curves for a particular type of soil. Input to this program consists of several parameters such as pile diameter, soil stress-strain relationship, unit weight of soil, internal friction angle if sand or cohesion if clay, and relative density if sand or consistency if clay. Once the p-y curves are determined and input into the pile analysis program, the pile stiffness coefficients can be determined from the output. The output is in the form of pile deflection and moment, soil modulus, and soil reaction values as a function of depth along the pile. The only major disadvantage of this program is that the user must calculate the pile stiffness coefficients from the output. The lateral stiffness coefficients, b_{11} and b_{22}, may be calculated using applied loads and pile head displacements (computer output); and the coupling coefficients, b_{42} and b_{51}, may likewise be calculated using the displacements and moments. Since the stiffness matrix must be symmetric, $b_{42} = b_{24}$ and $b_{15} = b_{51}$. The calculation of the rotational stiffness coefficients, b_{44} and b_{55}, is accomplished by specifying a slope (rotation) at the top of the pile and then using this slope and the outputted moments to determine b_{44} and b_{55}. Dr. Reese's programs have been adapted for the Corps' timesharing library. For documentation of these programs see reference 23. For an example using these programs to determine the pile stiffness coefficients, see the sample problem at the end of this appendix.

E. Summary and Recommendations. Calculation of the lateral pile stiffness coefficients for use in a pile foundation analysis can be accomplished by using data from pile load tests or by mathematically

analyzing a single pile. If soil parameters are not well defined, a "bracket" approach should be used for the analytical method to account for the numerous unknowns and assumptions involved. Results from the analysis of a single pile using one of the computer programs discussed can be used to verify the validity of the upper and lower limits of the "bracket". Another approach when soil parameters are sufficiently defined through testing, is to develop a set of stiffness coefficients for the anticipated loads using one of the programs discussed and then use these coefficients for the pile foundation analysis. In any case, where large numbers of pile are used or the subsurface conditions are out of the ordinary, analytical assumptions and results should be verified and/or modified by actual pile load tests.

III. AXIAL STIFFNESS.

A. General. The axial stiffness of a pile depends on many factors such as the modulus of elasticity of the pile, the pile area, the pile length, the pile tip deflection, the distribution of axial skin friction along the pile, and the percentage of axial load transmitted to the tip. Many of these factors are greatly affected by other related items such as type of pile hammer, level of the water table, soil density, etc. Some of the factors mentioned above, such as the pile length, area, and modulus of elasticity are easily determined while some of the others are more difficult to ascertain.

B. Tip Deflection and Distribution of Axial Forces Along the Pile. The pile tip deflection under load and the manner in which the axial force in the pile is transmitted to soil are interrelated and can have a great effect on the axial stiffness of the pile. Research has indicated (21 and 22) that the amount of load resisted by skin friction along the pile is dependent on

the amount of pile tip deflection. Predicting pile tip deflection accurately is very uncertain. Most of the group pile analyses to date have assumed that, for compressive loads, pile tip deflection under service loads is negligible. For this assumption, the axial stiffness can be assumed to be AE/L for an end bearing pile with no load resisted by skin friction and 2AE/L for a friction pile with no end bearing load transfer. These axial stiffnesses are analogous to column effective lengths of L and L/2. Based on a particular load distribution between end bearing and skin friction, the modifier for the axial stiffness could vary between one and two. It must be emphasized that the above discussion applies to pile with compressive loads and negligible tip deflection. Tests have shown that pile having tensile loads are less stiff (as much as a 50 percent reduction) than piles with a compressive load. Furthermore, deflection of the pile tip of a relatively small amount can cause the axial stiffness to be significantly different. Additional research needs to be done to more definitely predict the axial pile stiffness for piles with tip deflection. In the absence of better data, the values for axial pile stiffness shown in Table C-3 have been used by some designers.

A computer program developed by Drs. Lymon Reese and H.M. Coyle can be used to compute load-displacement relationships for axially loaded piles. The load transfer curves used in the program to relate skin friction to the axial displacement of the pile are based on semi-empirical criteria. For more information on this program see reference 23.

TABLE C-3

AXIAL PILE STIFFNESS COEFFICIENTS
(Assuming No Tip Deflection)

Condition	b_{33}
Compressive load, end bearing pile	$\frac{AE}{L}$
Compressive load, friction pile	$\frac{2AE}{L}$

IV. TORSIONAL STIFFNESS.

For a three dimensional pile group, a torsional pile constant (b_{66}) can be defined which relates the rotation of the pile in a plane perpendicular to its longitudinal axis to an applied torque. This can be expressed as

$$b_{66} = C_T \frac{JG}{L}$$

where:

- C_T is a constant which describes the distribution of torsional shear to the soil and the transfer of torsional shear resistance from the pile to the structure.
- J is the polar moment of inertia of the pile
- G is the shearing modulus of elasticity
- L is the length of pile

Unless the pile group is small (say less than 10 piles), the torsional stiffness of the individual pile appears to have little effect on the stiffness of the pile group and can be conservatively assumed to be zero.

EXAMPLE PROBLEM:

ANALYSIS OF SINGLE PILE TO DETERMINE PILE STIFFNESS COEFFICIENTS

Determine the pile stiffness coefficients for the pile shown in Figure C-2 using p-y curves to represent the lateral resistance of the soil. Assume the pile head to be fixed to the pile cap.

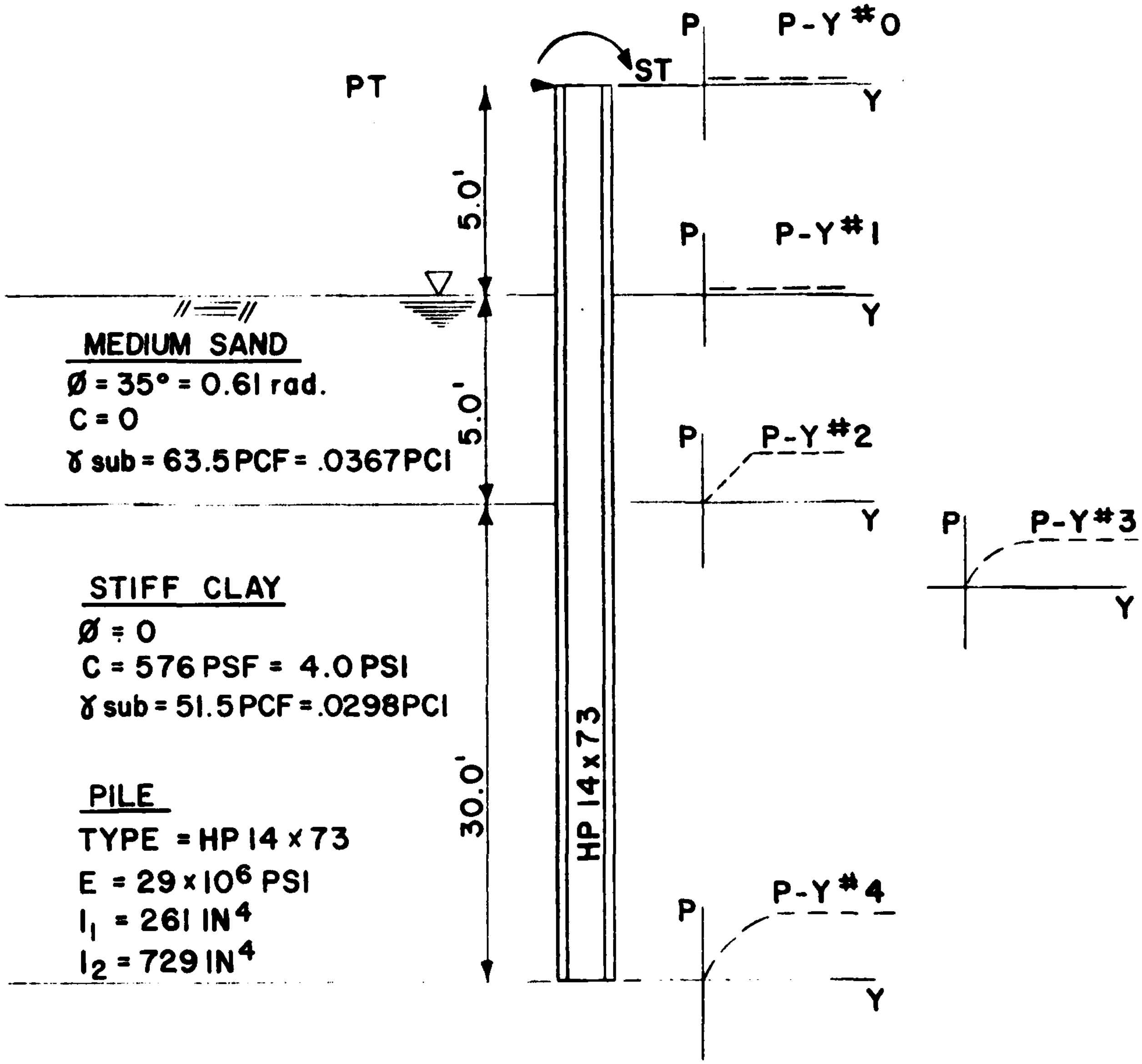


FIGURE C-2

STEP #1. Determine the p-y curves for the soil strata shown in Figure C-2. Use computer program CORPS/10004 (MAKE) to generate the p-y curve data. This program also has graphics capabilities which can be used to plot the input data and the p-y curves. Four p-y curves are generated: one at the ground surface, one at the bottom of the sand layer, one near the top of the clay layer, and one at the bottom of the clay layer. The plot of the input data for the example is shown in Figure C-3. Figures C-4, C-5, and C-6 are plots with different scales of the p-y curves developed by the program. Figure C-7 is a listing of the output file for the example. For a complete program description and variable definitions see reference 23.

FIGURE C-3: PLOT OF INPUT

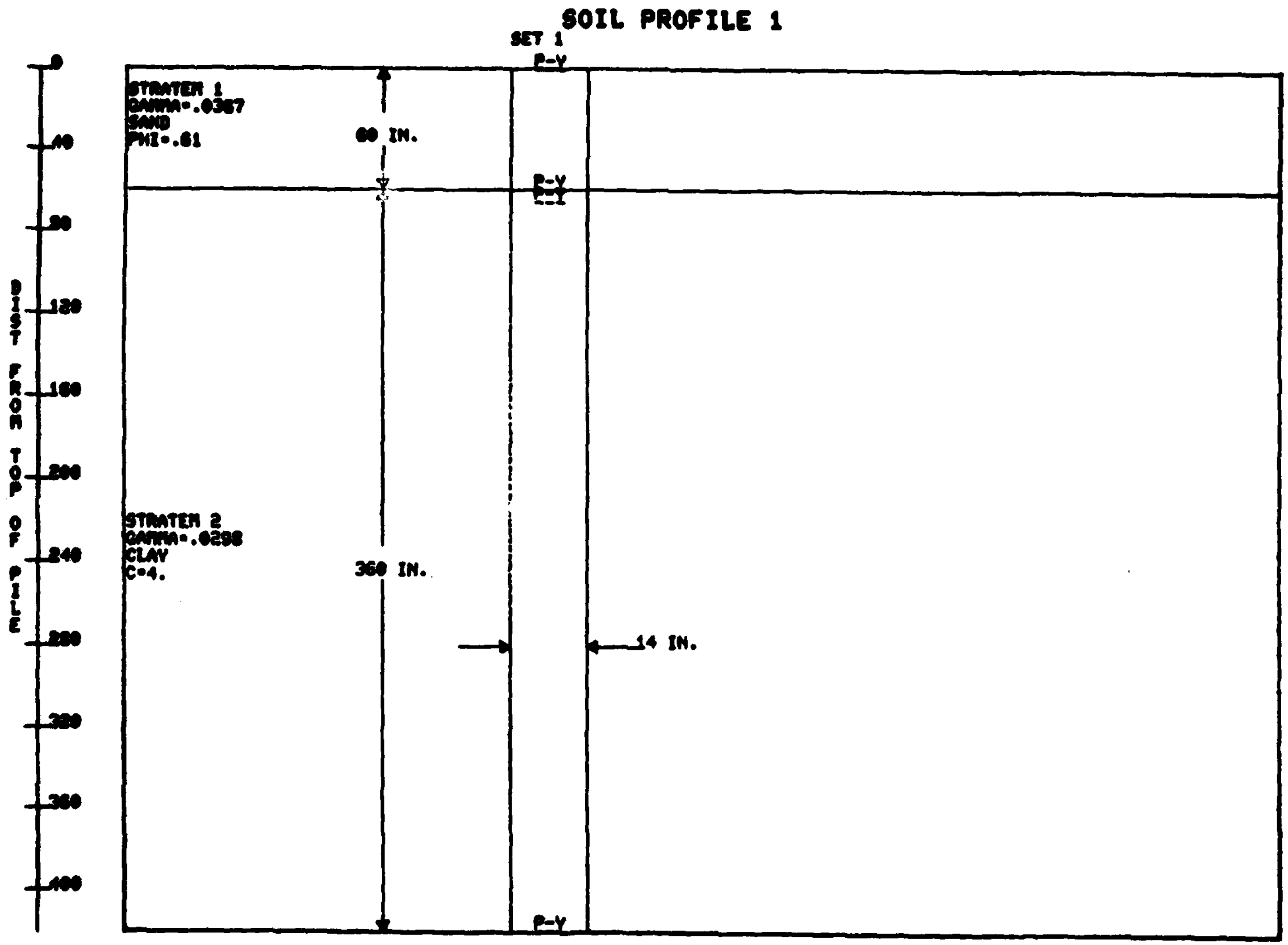


FIGURE C-4: PLOT OF OUTPUT

P-Y CURVE SET 1

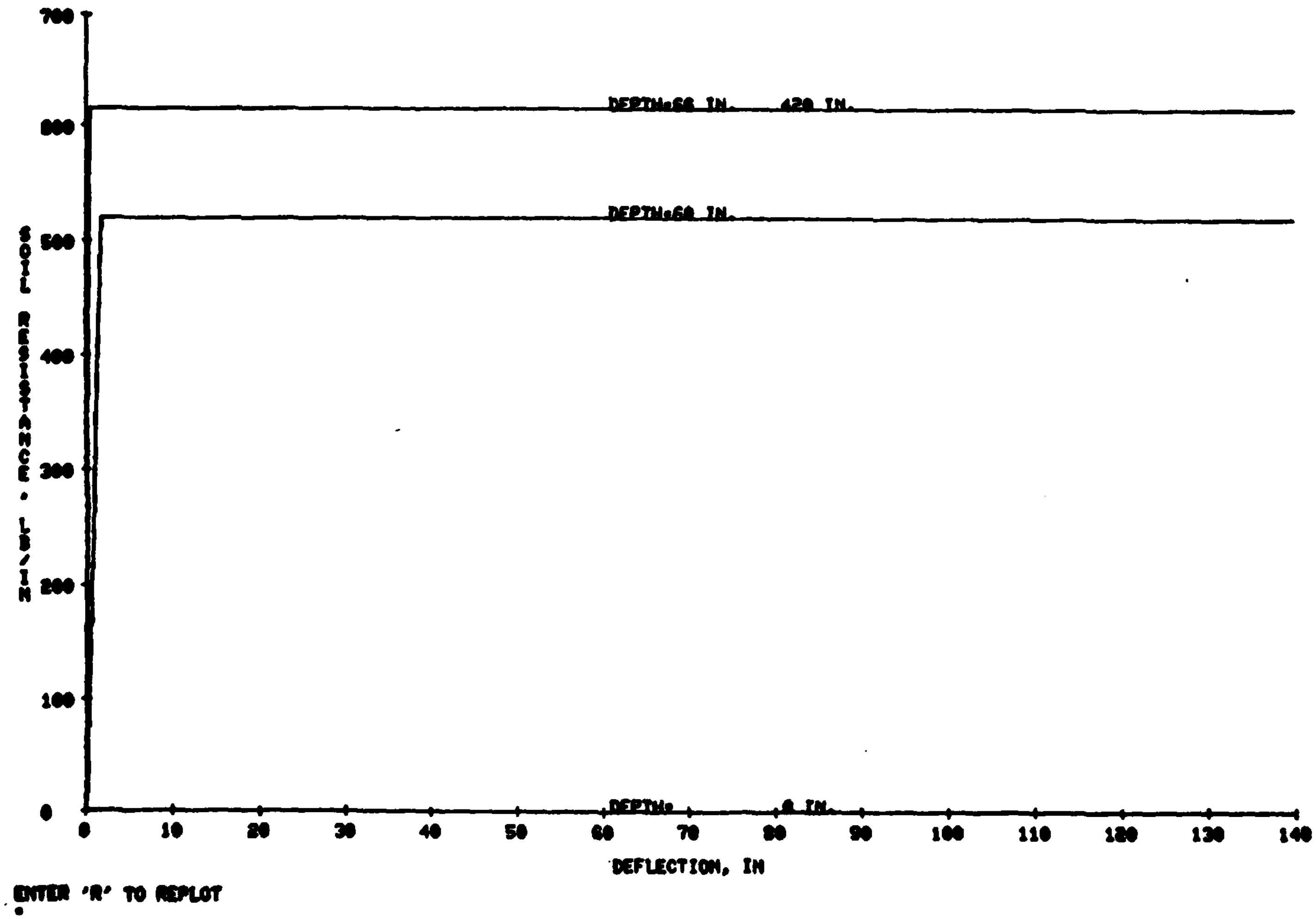


FIGURE C-5: PLOT OF OUTPUT

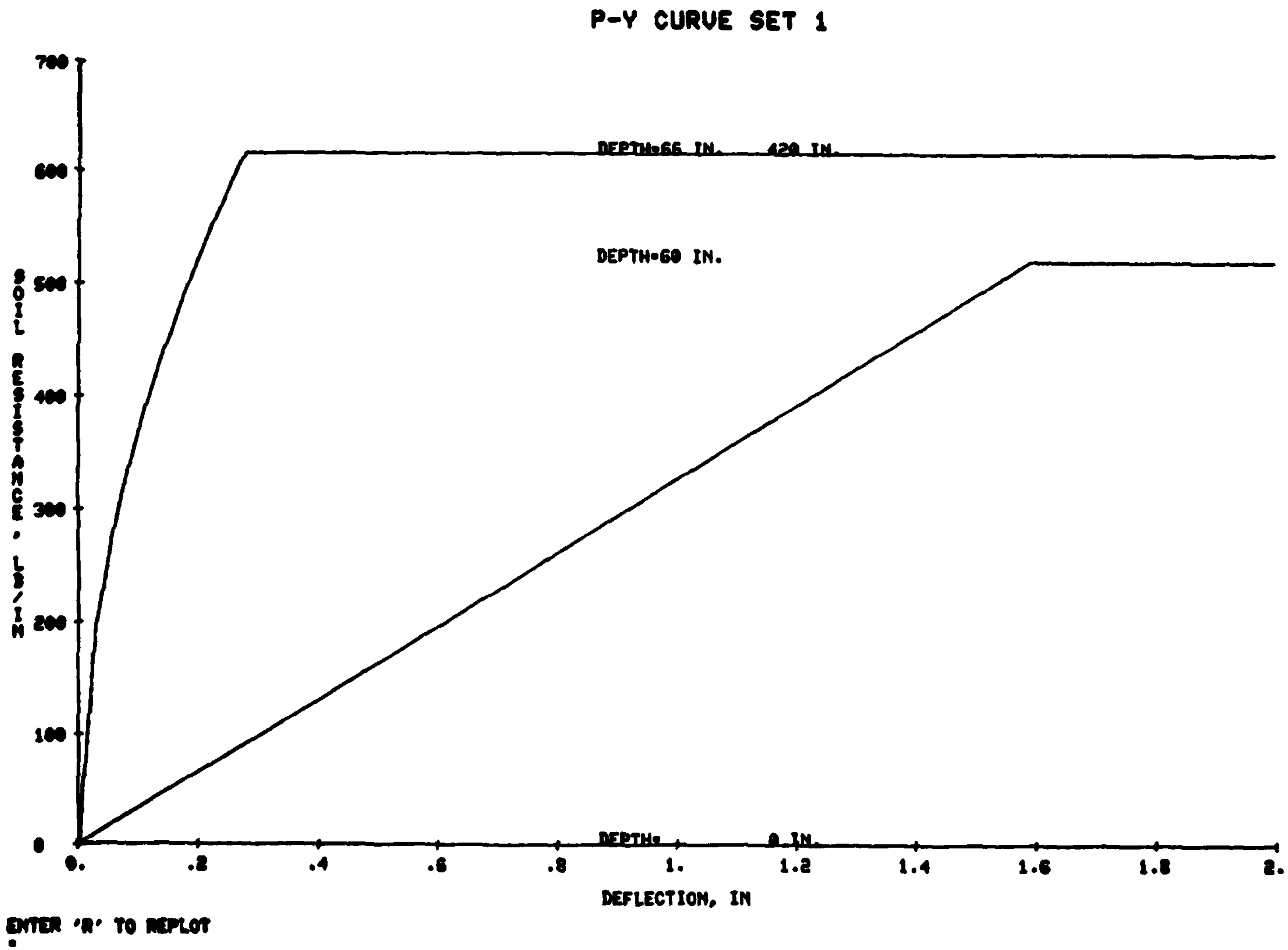


FIGURE C-6: PLOT OF OUTPUT

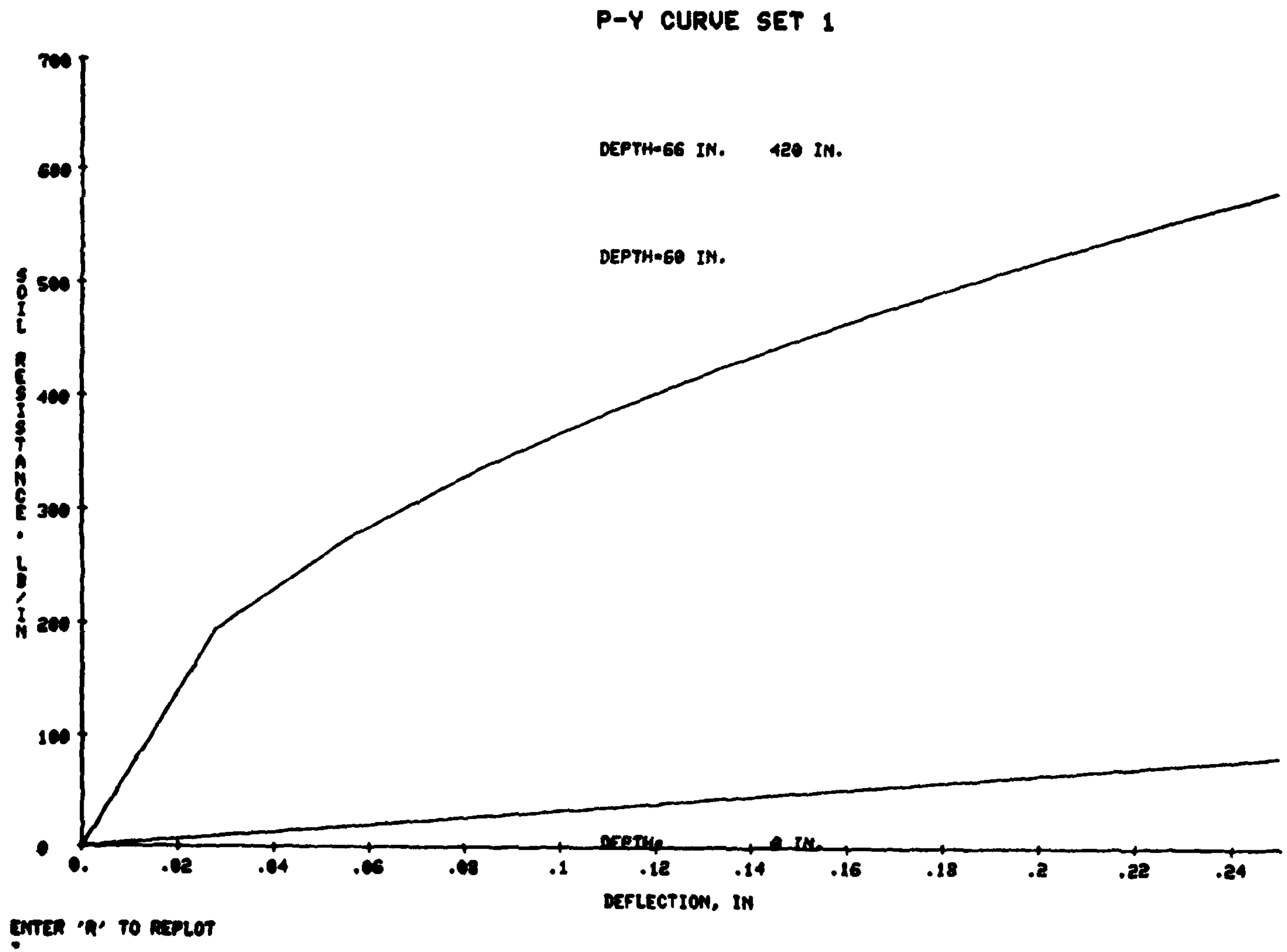


FIGURE C-7: LISTING OF OUTPUT

```
LIST PYOUT


     INPUT OF SOIL PARAMETERS


     SOIL PROFILE NO.     1    STRATUM NO.,    1 TYPE SOIL SAND

   UNIT         ANGLE OF            TOP           BOTTOM    DENSITY
  WEIGHT        FRIC.              DEPTH          DEPTH
 0.3670E-01   0.6100E 00          0.             0.6000E 02  MEDIU
     SOIL PROFILE NO.     1    STRATUM NO.,    2 TYPE SOIL CLAY

     UNIT   COHESION      TOP      BOTTOM          CONSISTENCY
    WEIGTH               DEPTH     DEPTH
 0.2980E-01       0.4000E 01      0.6000E 02      0.4200E 03     STIF


     PROPERTIES OF PILE USED FOR GENERATION OF PY-CURVES

 SET IDENTIFIER NO.     1     NUMBER OF CURVES IN SET      4


 DIAMETER DISTRIBUTION FOR PILE


    DIAMETER        TOP DIS        BOT DIS
  0.1400E 02     0.              0.4200E 03


 CURVE NO.     1 DEPTH TO CURVE 0.
 SOIL REACTION          DEFLECTION
    0.                  0.
    0.                  0.1000E 01
    0.                  0.1400E 03
```

CURVE NO. 2 DEPTH TO CURVE 0.6000E 02

SOIL REACTION	DEFLECTION
0.	0.
0.5201E 03	0.1594E 01
0.5201E 03	0.1400E 03

CURVE NO. 3 DEPTH TO CURVE 0.6600E 02

SOIL REACTION	DEFLECTION
0.	0.
0.1948E 03	0.2800E-01
0.2755E 03	0.5600E-01
0.3374E 03	0.8400E-01
0.3896E 03	0.1120E 00
0.4356E 03	0.1400E 00
0.4772E 03	0.1680E 00
0.5154E 03	0.1960E 00
0.5510E 03	0.2240E 00
0.5844E 03	0.2520E 00
0.6160E 03	0.2800E 00
0.6160E 03	0.1400E 03

CURVE NO. 4 DEPTH TO CURVE 0.4200E 03

SOIL REACTION	DEFLECTION
0.	0.
0.1948E 03	0.2800E-01
0.2755E 03	0.5600E-01
0.3374E 03	0.8400E-01
0.3896E 03	0.1120E 00
0.4356E 03	0.1400E 00
0.4772E 03	0.1680E 00
0.5154E 03	0.1960E 00
0.5510E 03	0.2240E 00
0.5844E 03	0.2520E 00
0.6160E 03	0.2800E 00
0.6160E 03	0.1400E 03

STEP #2. Once the p-y curves are determined, the force-displacement data for the curves are used to represent the soil behavior and are input to a program to analyze a single, laterally-loaded pile. For this analysis use program CORPS/I0001 (COM62). An additional p-y curve (#0) has been added at the top of the pile. This set of data merely represents the absence of soil at the top of the pile. Program COM62 uses a linear interpolation between succeeding p-y curves that are input. Using the p-y curves as specified force-deformation relations for the soil and the pile flexural stiffness, the program solves the differential equations representing the pile-soil relationship. This is performed in an iterative fashion until the elastic behavior of the pile matches the specified soil behavior. Four runs of computer program COM62 are required. Two runs are required to determine the respective pile stiffness coefficients for each principal axis of the pile. In the first run only a lateral force is applied along pile axis 1 to the top of the pile In the second run, a slope along the pile axis 1 is applied at the top of the pile. The graphics associated with program COM62 give plots of the input as well as the soil pressure, moments, and deflections for the pile. Plots of the input data for the first two runs are shown in Figures C-8 and C-9 and a plot of the output for the runs 1 and 2 are shown in Figures C-10 and C-11 (respectively). A listing of selected output is shown in Figure C-12. For a complete program description and variable definitions see reference 23.

FIGURE C-8: PLOT OF INPUT DATA

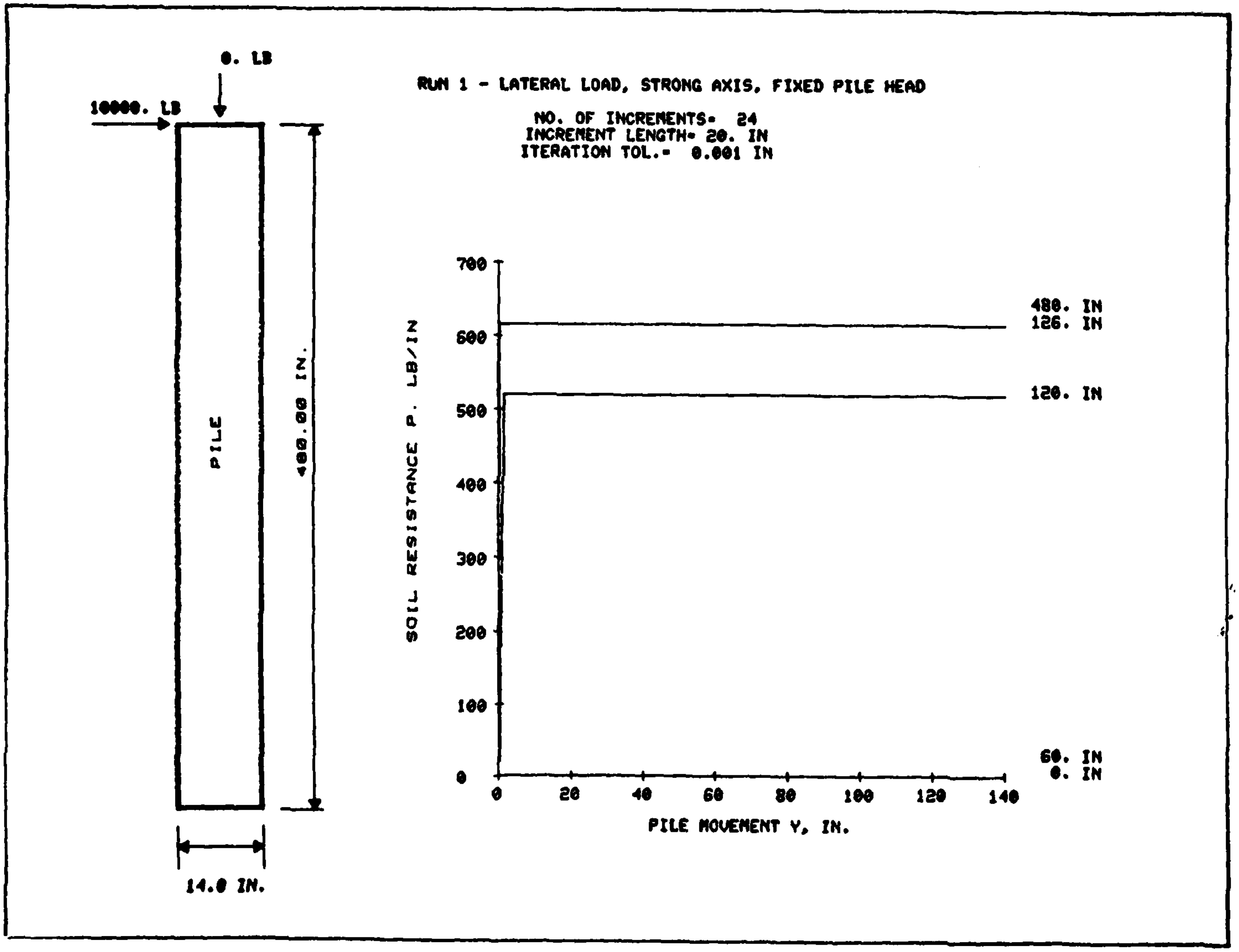


FIGURE C-9: PLOT OF INPUT DATA

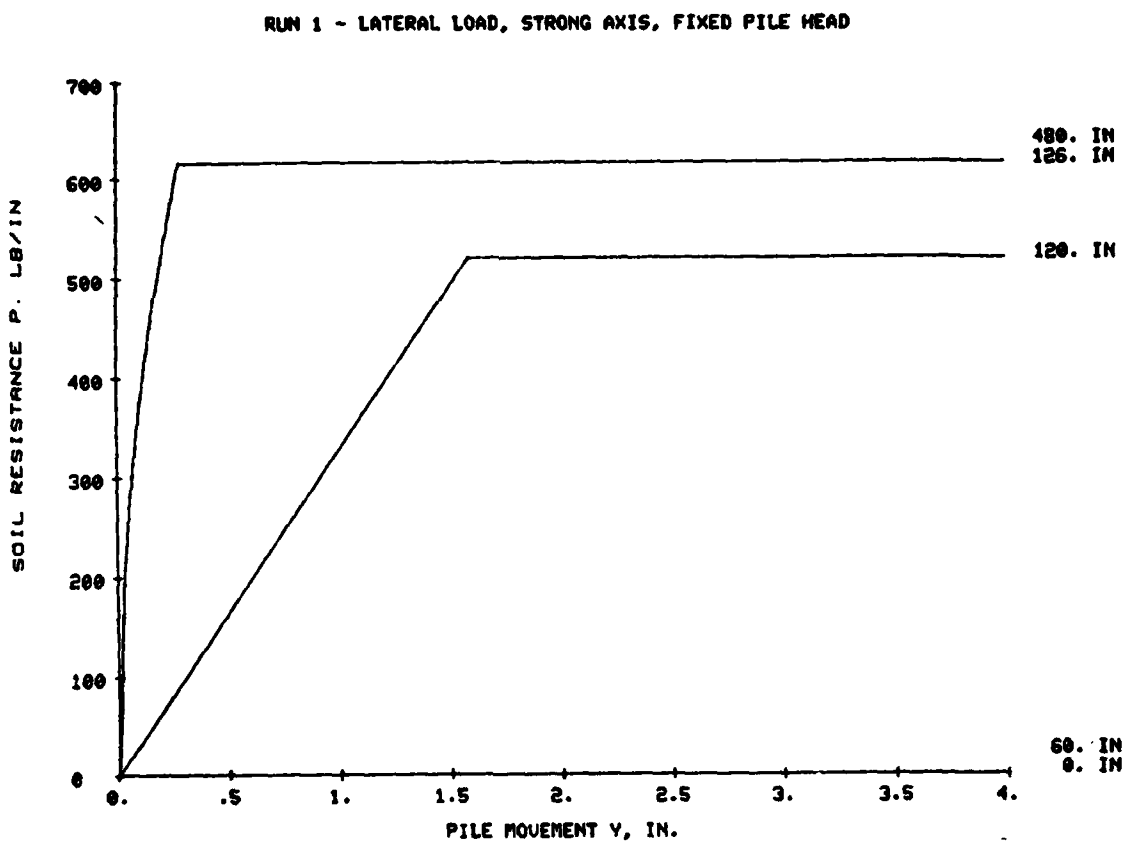


FIGURE C-10: PLOT OF OUTPUT FOR RUN 1

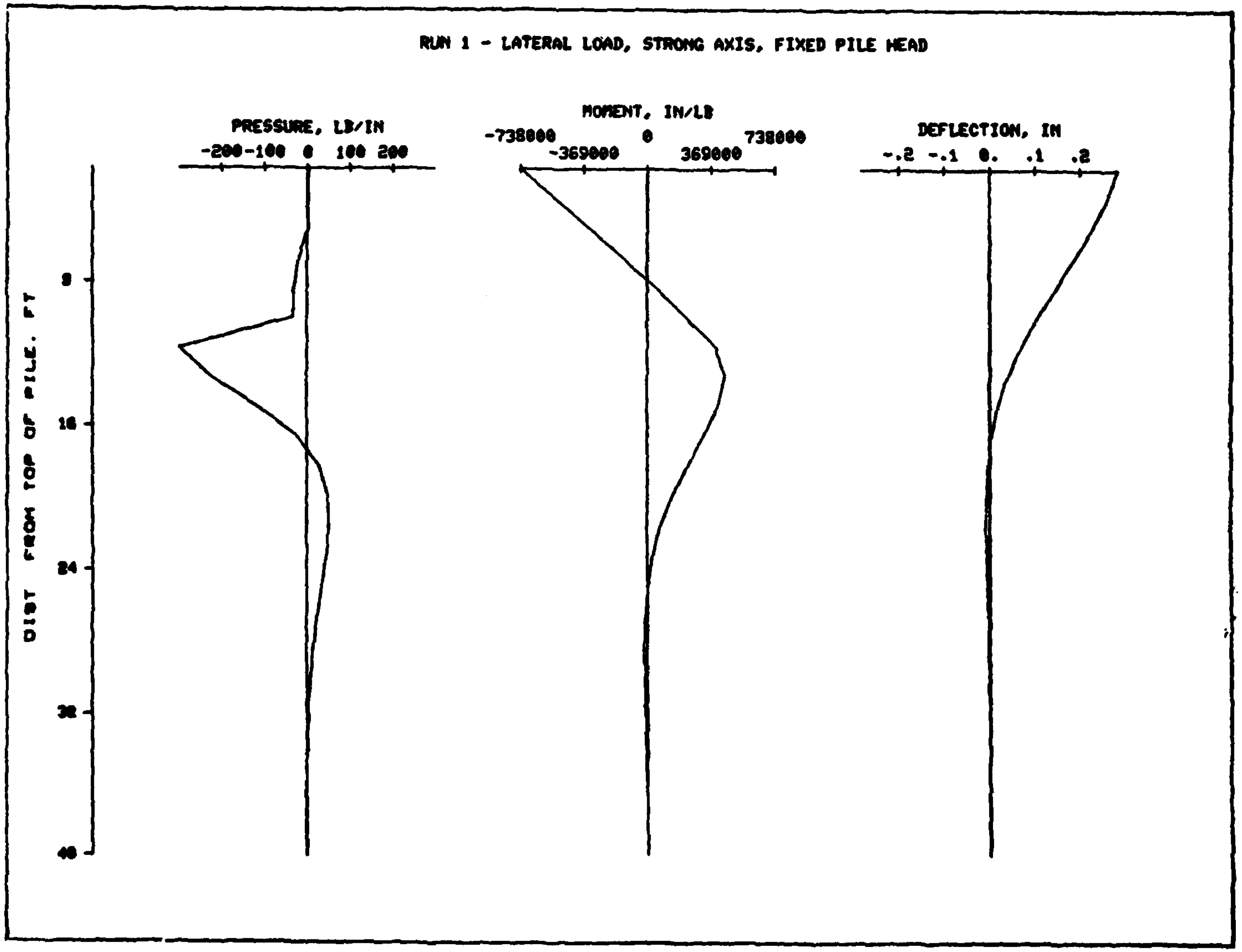


FIGURE C-11: PLOT OF OUTPUT FOR RUN 2

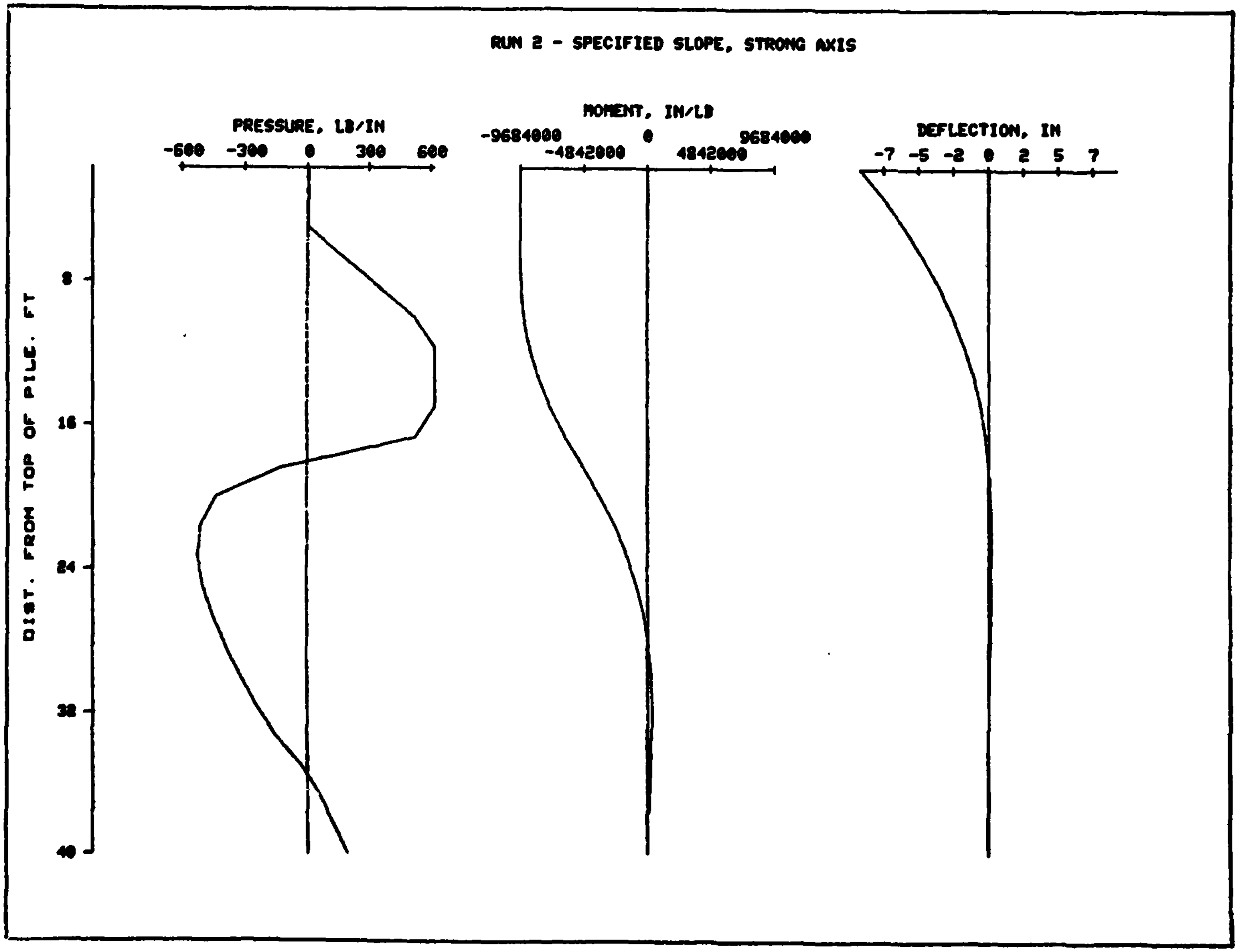


FIGURE C-12: LISTING OF SELECTED OUTPUT

RUN 1 - LATERAL LOAD, STRONG AXIS, FIXED PILE HEAD

```
OUTPUT INFORMATION
  X,FT.        Y,IN.        M,IN-LB    ES,LB/IN2  P,LB/IN.    EI,LB/IN2
0.           0.2919E 00-0.9385E 06 0.            0.          0.2100E 11
0.1667E 01 0.2830E 00-0.7385E 06 0.            0.          0.2100E 11
0.3333E 01 0.2600E 00-0.5385E 06 0.            0.          0.2100E 11
0.5000E 01 0.2267E 00-0.3385E 06 0.            0.          0.2100E 11
```

RUN 2 - SPECIFIED SLOPE, STRONG AXIS

OUTPUT INFORMATION

X,FT.	Y,IN.	M,IN-LB	ES,LB/IN2	P,LB/IN.	EI,LB/IN2
0.	-0.1114E 02	-0.9685E 07	0.	0.	0.2100E 11
0.1667E 01	-0.9232E 01	-0.9685E 07	0.	0.	0.2100E 11
0.3333E 01	-0.7508E 01	-0.9685E 07	0.	0.	0.2100E 11
0.5000E 01	-0.5969E 01	-0.9685E 07	0.	0.	0.2100E 11
0.6667E 01	-0.4615E 01	-0.9685E 07	0.3757E 02	0.1734E 03	0.2100E 11
0.8333E 01	-0.3445E 01	-0.9615E 07	0.1007E 03	0.3468E 03	0.2100E 11
0.1000E 02	-0.2459E 01	-0.9407E 07	0.2116E 03	0.5202E 03	0.2100E 11
0.1167E 02	-0.1651E 01	-0.8991E 07	0.3732E 03	0.6162E 03	0.2100E 11
0.1333E 02	-0.1015E 01	-0.8329E 07	0.6073E 03	0.6162E 03	0.2100E 11
0.1500E 02	-0.5371E 00	-0.7419E 07	0.1147E 04	0.6163E 03	0.2100E 11
0.1667E 02	-0.2008E 00	-0.6264E 07	0.2598E 04	0.5218E 03	0.2100E 11
0.1833E 02	0.1617E-01	-0.4899E 07	0.6946E 04	-0.1123E 03	0.2100E 11
0.2000E 02	0.1398E 00	-0.3580E 07	0.3112E 04	-0.4352E 03	0.2100E 11
0.2167E 02	0.1953E 00	-0.2435E 07	0.2634E 04	-0.5144E 03	0.2100E 11
0.2333E 02	0.2044E 00	-0.1495E 07	0.2573E 04	-0.5260E 03	0.2100E 11
0.2500E 02	0.1850E 00	-0.7657E 06	0.2704E 04	-0.5004E 03	0.2100E 11
0.2667E 02	0.1511E 00	-0.2366E 06	0.2992E 04	-0.4521E 03	0.2100E 11

STEP 3. Using the output from first run, the stiffness coefficients b_{11}, b_{51}, and b_{15} can be determined:

$$b_{11} = \frac{\text{applied lateral force along pile axis 1}}{\text{output displacement at top of pile}} = \frac{10{,}000 \text{ lbs.}}{0.2919 \text{ in.}} = 34{,}258 \text{ lb/in}$$

$$b_{51} = \frac{\text{output moment at top of pile}}{\text{output displacement at top of pile}} = \frac{938{,}500 \text{ in-lb}}{0.2919 \text{ in}} = 3.215\text{x}10^6 \text{ in-lb/in}$$

$$b_{15} = b_{51}$$

Using the output information from second run, the stiffness coefficient b_{55} can be determined:

$$b_{55} = \frac{\text{output moment at top of pile}}{\text{applied rotation (slope)}} = \frac{9.685\text{x}10^6 \text{ in-lb}}{0.10 \text{ rad}} = 9.685\text{x}10^7 \frac{\text{in-lb}}{\text{rad}}$$

The stiffness constants b_{22}, b_{24}, b_{42}, and b_{44} can be similarly obtained by making a third and fourth run of COM62 with the lateral force and slope applied with respect to the other pile axis. These values for the sample problem are:

$$b_{22} = 16{,}113 \text{ lb/in}$$

$$b_{24} = 1.385\text{x}10^6 \frac{\text{in-lb}}{\text{in}}$$

$$b_{44} = 4.236\text{x}10^7 \frac{\text{in-lb}}{\text{rad}}$$

$$b_{42} = b_{42}$$

The stiffness constants b_{33} (axial) and b_{66} (torsional) are calculated by other means (see sections III and IV of this appendix).

APPENDIX D

SOIL MODULUS FOR LATERALLY LOADED PILES

The lateral pile stiffness coefficients discussed in Appendix C can be directly computed when E_s is a constant or linearly varying in the media in which the pile is embedded. It is generally assumed that for homogeneous cohesive soils E_s is constant and for homogeneous cohesionless soils E_s varies linearly with depth. In this Appendix, typical values for computing E_s for homogeneous cohesive and cohesionless soils are provided. The structural engineer must rely on the geotechnical engineer to obtain the values of the soil modulus and the coefficient of horizontal subgrade reaction.

Definitions and Nomenclature

K_1	- Coefficient of horizontal subgrade reaction (lbs/ft^3) (ratio of pressure (lbs/ft^2) at a point to the displacement (ft) at the point) for a 1 foot wide pile embedded in clay.
K_h	- Coefficient of horizontal subgrade reaction (lbs/ft^3) (K_h = $2K_1$(1ft)/B(ft)).
E_s	- Soil modulus (lbs/ft^2) - ratio of soil resistance (p) (lbs/ft) to pile movement (y) (ft), $E_s = K_h B$).
n_h	- Constant of horizontal subgrade reaction (lbs/ft^3) for a pile 1 foot wide embedded in sand ($K_h = n_h Z/B$, $E_s = n_h z$).
B	- Width of pile (ft).
Z	- Depth below ground surface (ft).
q_u	- Unconfined compressive strength of clay (lbs/ft^2).
R_1 & R_2	- Reduction factors
γ	- Unit weight of soil

Homogeneous Soils

If the soil is homogeneous and can be classified as clay (cohesive) or sand (cohesionless), estimates of E_s can be computed using values developed by Terzaghi (11), Broms (9, 10), and others based on experiments and theoretical relations. The range of values provided by these authors (see next sections) must be reduced for cyclic and group effects on piles. It should be noted that these values are based on the assumption that the soil has linear elastic properties. The values are only valid for simple soil conditions and must be used with caution.

Cohesive Soils

For cohesive soils E_s is assumed to be constant with depth.

$$E_s = K_h B \qquad (1)$$

$$K_h = K_1 \text{ (1ft/B)} \qquad (2)$$

The value of K_1 can be estimated by using the relation,

$$K_1 - a(80q_u) \qquad (3)$$

where a is a parameter on a 1 foot strip that varies from 0.32 to 0.52 (Reference 10, 20), generally use $a = 0.4$ (ft^{-1}).

q_u is the unconfined compression strength of clay in lbs/ft^2.

Thus,

$$K_1 = 0.4(80q_u) = 32q_u \qquad (4)$$

$$\text{Therefore, } K_h = \frac{32\ q_u}{B} \qquad (5)$$

Note that the units of K_h are in lbs/ft^3 with q_u being in lbs/ft^2 and B in feet.

$$E_s = 32q_u \qquad (6)$$

Note that the units of E_s in the above equation are the same as those of q_u.

Reduction Factors

The values obtained for E_s must be reduced to account for the effects of cyclic loading (R_1) and group action (R_2). Thus,

$$(E_s)_{adj} = (R_1)(R_2)E_s \qquad (7)$$

Use R_1 = 1 for initial loading and = 0.3 for cyclic loading (References 9, 10). The value of R_2 can be obtained from Table D-1 (Reference 1) given below.

TABLE D-1

Values of Group Effect Factor (R_2)

R_2	Pile Spacing in Direction of Loading
1.00	8D
0.85	7D
0.70	6D
0.55	5D
0.40	4D
0.25	3D

Cohesionless Soils

In cohesionless soils, due to confinement effects, E_s inceases with depth. It is generally assumed that this variation is linear.

$$E_s = n_h Z \qquad (8)$$

when n_h is the constant of horizontal subgrade reaction and Z is the depth below equivalent ground surface.

Terzaghi (Reference 11) provided the following empirical relation for obtaining n_h:

$$n_h = \frac{A\gamma}{1.35} \qquad (9)$$

where A is a value obtained as a function of the relative density of soil as shown in Table D-2 and γ is the effective unit weight of sand. The units of A will be the same as those of γ.

TABLE D-2

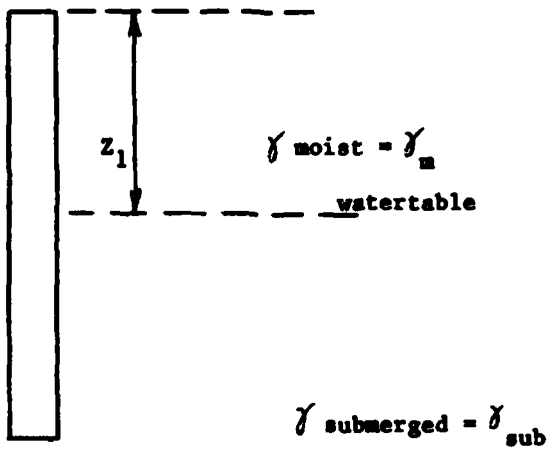
Values of A as a Function of the Relative Density of Sand

Relative Density	N*	Value of A
Loose	4 - 10	100 - 300
Medium	10 - 30	300 - 1000
Dense	30 - 50	1000 - 2000

*N is the number of blows of the drop weight required to drive the sampling spoon into the soil for a distance of one foot. A weight of 140 lbs and a height of fall of 30 inches are considered standard.

The value γ is the effective unit weight of the sand. If the piles are below the water table, the submerged unit weight of the sand should be used in computing the value of n_h. If the piles extend above the below the water table, an equivalent height of submerged sand should be developed above the water table. The depth Z then must be measured from the top of the equivalent ground surface. Thus,

$$Z_{eff} = \frac{\gamma_m}{\gamma_{sub}} \times Z_1 \qquad (10)$$

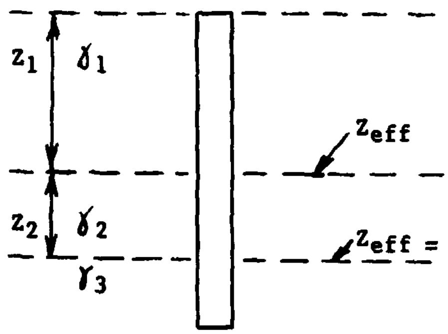


Reduction factors for cycle loading (R_1) and group loading (R_2) are applied in the same manner for cohesive soils.

Layered or Heterogeneous Soils

In general, for layered or heterogeneous soil deposits, E_s can vary arbitrarily with depth and the variation is difficult to be represented in a mathematical form. For these cases, p-y curves need to be developed along the length of the pile and computer programs as explained in Appendix C, Section II, C.2 can be used to compute the pile stiffness coefficients. The p-y curves can be determined based on laboratory triaxial tests or by measurements of the behavior of instrumented piles. References 8, 20 and 23 explain the development of p-y curves.

For homogeneous layered soils that are cohesive or cohesionless, the appropriate relations given in the preceding sections could be used. For cohesive soils, the E_s values are only dependent on the unconfined compressive strength of each of the layers. For cohesionless soils, the E_s values are dependent on n_h for the layer and the effective depth (Z_{eff}). The effective depth is calculated for each layer as shown:



$$Z_{eff} = \frac{\gamma_1 Z_1}{\gamma_2}$$

$$Z_{eff} = \frac{\gamma_1 Z_1 + \gamma_2 Z_2}{\gamma_3}$$

Summary

Typical values for E_s are given for homogeneous cohesive and cohesionless soils. It must be emphasized that the pile lateral stiffness coefficients can be directly computed only when E_s is a constant (such as for a homogeneous cohesive soil) or linearly varying (as for homogeneous cohesiveless soil). In all other cases including layered soils and soils that cannot be classified as cohesive or cohesionless, the pile stiffness coefficients can be calculated by using computer programs such as those elaborated in Appendix C, Section II, C.2 of this report.

APPENDIX E

EXAMPLE PROBLEMS

EXAMPLE PROBLEMS 1 THROUGH 5

Two-dimensional Problems

Hrennikoff's Example

Comparison of Computer Output with the Elastic Center Method

Hrennikoff's calculations and a Common Analytical Method

Example problems 1 through 5 illustrate how the computer can be used to analyze two-dimensional pile foundation problems. The following examples were taken from Hrennikoff's paper entitled "Analysis of Pile Foundations with Batter Piles", published in the Transactions of the American Society of Civil Engineers, Vol. 76, No. 1, Paper No. 2401, Jan. 1950, pp. 123-126.

The computer results are compared with Hrennikoff's results as well as with other hand computation methods commonly used by civil engineers.

The physical pile layout for example problems 1 through 5 is shown in Figure E1.

Example Problem No. 1 compares the results obtained by the Computer Method with those obtained by the Elastic Center Method assuming the soil offers no lateral support; in other words, the subgrade modulus is zero. The Computer results agree closely with the Elastic Center Method results. A description of the Elastic Center Method can be found in "Substructure Analysis and Design" by Dr. Paul Andersen. This method is limited, however, to pile groups consisting of hinged piles arranged in two subgroups whose centerlines intersect.

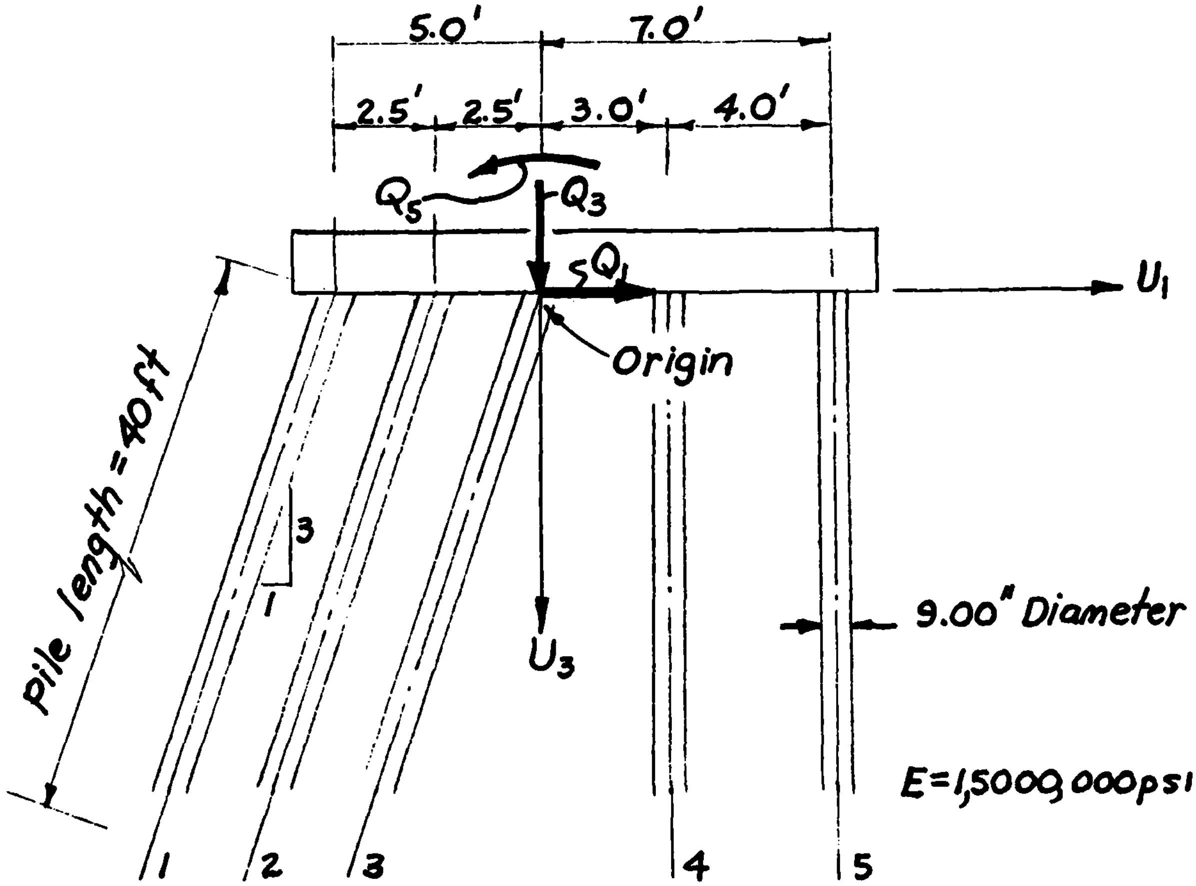


Figure E1. Physical problem for examples 1 through 5

Example Problem No. 2 compares the results obtained by the Computer Method with the results obtained by Manual Calculations as presented in Hrennikoff's paper for case 2a (very weak soil with a subgrade modulus of 3.123 psi). The Computer results agree closely with Hrennikoff's calculations.

Example Problem No. 3 compares the results obtained by the Computer Method with the results obtained by Manual Calculations as presented in Hrennikoff's paper for Case 4a (weak soil with a subgrade modulus of 31.23 psi). The Computer results agree closely with Hrennikoff's calculations.

Example Problem No. 4 compares the results obtained by the Computer Method with the results obtained by Manual Calculations as presented in Hrennikoff's paper for Case 6a (medium soil with a subgrade modulus of 312.3 psi). Again, the Computer results agree closely with Hrennikoff's calculations.

Example Problem No. 5 compares the results obtained by the Computer Method with the results obtained by a common Analytical Method for two different load conditions. A description of the common Analytical Method can be found in "Foundation Engineering" by Ralph Peck, Walter E. Hanson, and Thomas H. Thornburn, and in "Foundation Design" by Wayne Teng. Example Problem No. 5 demonstrates that the individual pile forces obtained by the common Analytical Method are approximate and may or may not agree closely with the results obtained by the Computer Method. A subgrade modulus of 312 psi was used for this example.

Example Problem 1

Two-dimensional problem
Hrennikoff's example
no lateral soil support

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
$Mod_{sub} = 0.1 \approx 0$	Pile resistance = 0.5
$I_x = 322.06$ in.^{4}	Participation factor for torsion = 0.0
$I_y = 322.06$ in.^{4}	Torsion modulus = 0.0
Area = 63.5 in.^{2}	
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Properties and loading conditions
for example problem 1

```
1 EXAMPLE PROBLEM NO 1
2 HRENNIKOFF EXAMPLE
3 2
4 5 1 1
5 1 .1
6 1 5 30.000 3
7 9.000
8 4
9 1500000.000
10 2
11 0. 1.0 0. 0.
12 1
13 82.000 40.000
14 2
15 1 3 -3.000
16 4 5 0.
17 -5.000 -2.5  0. 3.000 7.000
18 -39.375 113.1 173.4
E>FILE
STROM5 EDITED.
C>REPLACE,STROM5
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034

JOB WAITING.


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.



I>STROM5
```

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.	CONTENTS
1	PILE AND SOIL DATA
2	PILE COORDINATES AND BATTER
3	STIFFNESS AND FLEXIBILITY MATRICES FOR THE STRUCTURE AND COORDINATES OF ELASTIC CENTER
4	APPLIED LOADS
5	STRUCTURE DEFLECTIONS
6	PILE DEFLECTIONS ALONG PILE AXIS
7	PILE FORCES ALONG PILE AXIS
8	PILE FORCES ALONG STRUCTURE AXIS

```
INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.


I>


 INPUT A FILE NAME FOR OUTPUT IN 7 CHARACTERS OR LESS.
 HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.


I>


 EXAMPLE PROBLEM NO 1
 HRENNIKOFF EXAMPLE


 NO. OF PILE ROWS =   5    B MATRIX IS CALCULATED FOR EACH ROW


***************************************************************************
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
    1   5  E =  .15E+07 PSI   IX =   322.06 IN**4   IY =   322.06 IN**4
           AREA =   63.6 IN**2   X =      9.00 IN     Y =      9.00 IN
           LENGTH =   30.0 FEET   ES =    .100
           K1 =    .4107  K2 =   1.0000  K3 =   0.0000
           K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

LENGTH OF PILES (30.00 FEET) IS INSUFFICIENT
FOR PILE GROUP -   1 MINIMUM ACCEPTABLE LENGTH IS   87.88 FEET
FOR SEMI-INFINITE BEAM ON ELASTIC FOUNDATION

```
        ALLOWABLES:  COMPRESSIVE LOAD =      82.000 KIPS
                     TENSILE LOAD =     40.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   5 IS

```
  .186E+02  0.         0.
 0.          .265E+06  0.
 0.         0.         0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-3.00	-5.000
2	-3.00	-2.500
3	-3.00	0.000
4	VERTICAL	3.000
5	VERTICAL	7.000

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.796E+05	-.239E+06	-.716E+07
-.239E+06	.125E+07	-.103E+08
-.716E+07	-.103E+08	.329E+10

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.127E-03	.274E-04	.363E-06
.274E-04	.671E-05	.807E-07
.363E-06	.807E-07	.135E-08

COORDINATES OF ELASTIC CENTER
EC1 = .003 EC2 = -.002

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-39.375	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.116E+01	-.150E+00	-.236E-02

##

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.119E+01	.890E-01	-.236E-02
2	-.117E+01	.156E+00	-.236E-02
3	-.115E+01	.223E+00	-.236E-02
4	-.116E+01	-.655E-01	-.236E-02
5	-.116E+01	.478E-01	-.236E-02

##

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-.022	23.589	0.000	F
2	-.022	41.394	0.000	F
3	-.021	59.199	0.000	F
4	-.022	-17.359	0.000	
5	-.022	12.670	0.000	

TOTAL NO. FAILURES = 3 LOAD CASE 1

##

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-7.481	22.371	0.000
2	-13.111	39.263	0.000
3	-18.741	56.155	0.000
4	-.022	-17.359	0.000
5	-.022	12.670	0.000

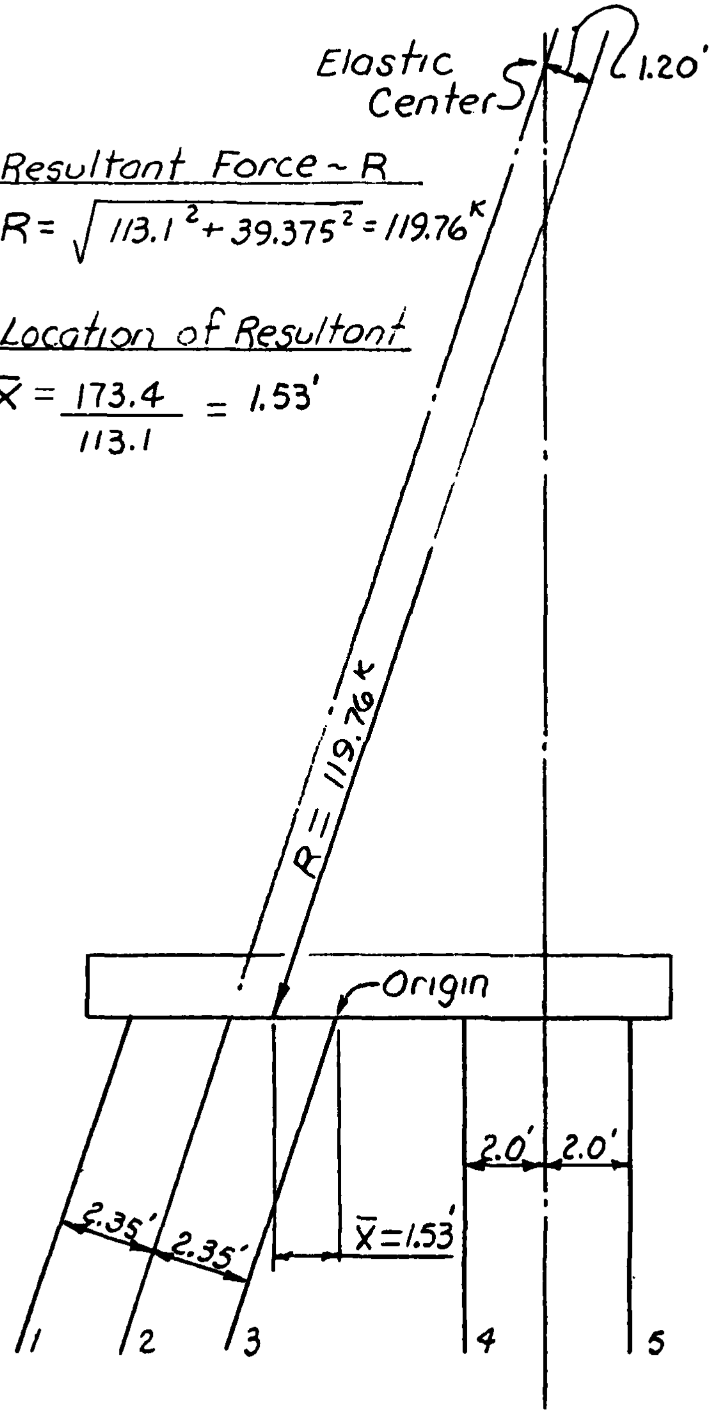
Analysis by elastic center method

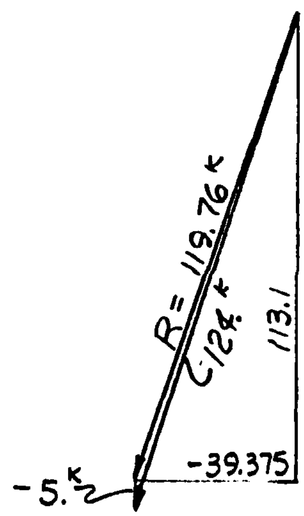
Resultant Force ~ R

$$R = \sqrt{113.1^2 + 39.375^2} = 119.76^K$$

Location of Resultant

$$\bar{X} = \frac{173.4}{113.1} = 1.53'$$





Force Diagram
Forces resolved in direction of pilegroups

Consider Piles 1, 2 & 3 are pile group A
and Piles 4 & 5 are pile group B

Then by the elastic center method,

$$P_A = \frac{R_A}{n_A} + \frac{r}{\Sigma r^2} M$$

and $$P_B = \frac{R_B}{n_B} + \frac{r}{\Sigma r^2} M$$

where: R_A & R_B are force components in directions of A & B pile groups

n_A & n_B are number of piles in A & B pile groups

r is the distance from the elastic center to the pile

M is the moment of the applied load about the elastic center.

$$\Sigma r^2 = 2(2.35)^2 + 2(2.0)^2 = 19.05$$

$$\text{Pile 1} = \frac{124}{3} - \frac{1.2(119.76)\,2.35}{19.05} = 23.61^K$$

$$\text{Pile 2} = \frac{124}{3} = 41.33^K$$

$$\text{Pile 3} = \frac{124}{3} + \frac{1.2\ (119.76)\,2.35}{19.05} = 59.06^K$$

$$\text{Pile 4} = \frac{-5}{2} - \frac{1.2(119.76)2}{19.05} = -17.59^K$$

$$\text{Pile 5} = \frac{-5}{2} + \frac{1.2\,(119.76)2}{19.05} = 12.59^K$$

Comparison of results obtained by the computer method with the results obtained by the elastic center method.

Pile Forces along Pile Axis

Pile No.	Computer Output F_3 (Kips)	Elastic Center F_3 (Kips)
1	23.589	23.61
2	41.394	41.33
3	59.199	59.06
4	-17.359	-17.59
5	12.670	12.59

Example Problem 2

Two-dimensional problem,
Hrennikoff's example
case 2a (very weak soil)

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
$Mod_{sub} = 3.123$ psi	Pile resistance = 0.5
$I_x = 322.06$ in.^{4}	Participation factor for torsion = 0.0
$I_y = 322.06$ in.^{4}	Torsion modulus = 0.0
Area = 63.5 in.^{2}	
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Properties and loading conditions for example problem 2

```
1 EXAMPLE PROBLEM NO 2
2 HRENNIKOFF EXAMPLE
3 2
4 5 1 1
5 1 3.123
6 1 5 30.000 3
7 9.000
8 4
9 1500000.000
10 2
11 0. .5 0. 0.
12 1
13 82.000 40.000
14 2
15 1 3 -3.000
16 4 5 0.
17 -5.000 -2.5  0. 3.000 7.000
18 -39.375 113.1 173.4
E>FILE
STROM5 EDITED.
C>REPLACE,STROM5
C>OLD,CORPS,
 ERROR IN ARGUMENT.
C>OLD,CORPS/UN=CECEL3
C>CALL,CORPS,X0034


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.


I>STROM5



 THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                  CONTENTS
    1              PILE AND SOIL DATA
    2              PILE COORDINATES AND BATTER
    3              STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                   STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4              APPLIED LOADS
    5              STRUCTURE DEFLECTIONS
    6              PILE DEFLECTIONS ALONG PILE AXIS
    7              PILE FORCES ALONG PILE AXIS
    8              PILE FORCES ALONG STRUCTURE AXIS
```

```
INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.
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```
EXAMPLE PROBLEM NO 2
HRENNIKOFF EXAMPLE

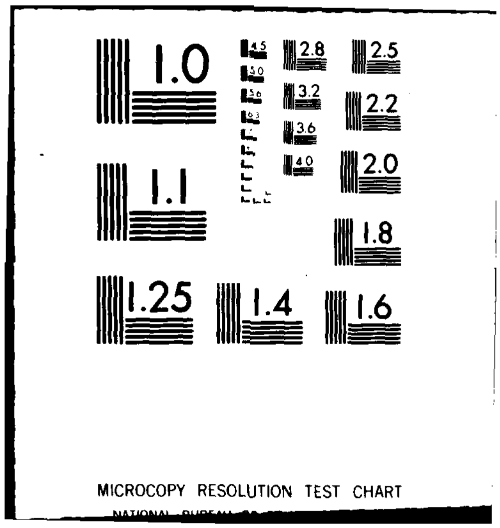

NO. OF PILE ROWS =   5    B MATRIX IS CALCULATED FOR EACH ROW


********************************************************************************
```

AD-A093 696 ARMY ENGINEER WATERWAYS EXPERIMENT STATION VICKSBURG MS F/6 13/13
BASIC PILE GROUP BEHAVIOR.(U)
DEC 80 J HARTMAN
UNCLASSIFIED WES-TR-K-80-5 NL

2 of 2
AD A
093696

END
DATE
FILMED
2-81
DTIC


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
1   5  E =  .15E+07 PSI   IX =   322.06 IN**4   IY =   322.06 IN**4
       AREA =   63.6 IN**2   X =      9.00 IN      Y =      9.00 IN
       LENGTH =   30.0 FEET   ES =   3.123
       K1 =    .4107  K2 =    .5000  K3 =   0.0000
       K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

LENGTH OF PILES (30.00 FEET) IS INSUFFICIENT
FOR PILE GROUP -   1 MINIMUM ACCEPTABLE LENGTH IS   37.17 FEET
FOR SEMI-INFINITE BEAM ON ELASTIC FOUNDATION

```
ALLOWABLES:  COMPRESSIVE LOAD =      82.000 KIPS
             TENSILE LOAD =     40.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   5 IS

```
 .246E+03  0.         0.
0.         .133E+06  0.
0.        0.          0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-3.00	-5.000
2	-3.00	-2.500
3	-3.00	0.000
4	VERTICAL	3.000
5	VERTICAL	7.000

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.409E+05	-.119E+06	-.357E+07
-.119E+06	.623E+06	-.517E+07
-.357E+07	-.517E+07	.164E+10

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.193E-03	.414E-04	.550E-06
.414E-04	.105E-04	.123E-06
.550E-06	.123E-06	.219E-08

COORDINATES OF ELASTIC CENTER
EC1 = .003 EC2 = -.002

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-39.375	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.177E+01	-.183E+00	-.315E-02

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.180E+01	.207E+00	-.315E-02
2	-.177E+01	.296E+00	-.315E-02
3	-.174E+01	.386E+00	-.315E-02
4	-.177E+01	-.692E-01	-.315E-02
5	-.177E+01	.821E-01	-.315E-02

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU	CO	TE
1	-.442	27.395	0.000	F		
2	-.435	39.282	0.000	F		
3	-.427	51.170	0.000	F		
4	-.436	-9.167	0.000			
5	-.436	10.881	0.000			

TOTAL NO. FAILURES = 3   LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-9.082	25.849	0.000
2	-12.835	37.129	0.000
3	-16.587	48.408	0.000
4	-.435	-9.167	0.000
5	-.436	10.381	0.000
SUM	-39.375	113.190	173.400

****

Pile No.	Computer Output F1 (kips)	Computer Output F3 (kips)	Hrennikoff's Example F1 (kips)	Hrennikoff's Example F3 (kips)
1	0.442	27.395	0.44	27.5
2	0.435	39.282	0.43	39.3
3	0.427	51.170	0.43	51.0
4	0.436	-9.167	0.43	-9.0
5	0.436	10.881	0.43	10.9

Example Problem 3

Two-dimensional problem,
Hrennikoff's example
case 4a (weak soil)

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
Mod_{sub} = 31.230 psi	Pile resistance = 1.0
I_x = 322.06 $in.^4$	Participation factor for torsion = 0.0
I_y = 322.06 $in.^4$	
Area = 63.5 $in.^2$	Torsion modulus = 0.0
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Properties and loading conditions for example problem 3

```
1 EXAMPLE PROBLEM NO 3
2 HRENNIKOFF EXAMPLE
3 2
4 5 1 1
5 1 31.23
6 1 5 30.000 3
7 9.000
8 4
9 1500000.000
10 2
11 0. 1.0 0. 0.
12 1
13 82.000 40.000
14 2
15 1 3 -3.000
16 4 5 0.
17 -5.000 -2.5  0. 3.000 7.000
18 -39.375 113.1 173.4
E>FILE
STROM5 EDITED.
C>REPLACE,STROM5
C>OLD,CORPCS
 ERROR IN ARGUMENT.
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.



I>STROM5
```

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.	CONTENTS
1	PILE AND SOIL DATA
2	PILE COORDINATES AND BATTER
3	STIFFNESS AND FLEXIBILITY MATRICES FOR THE STRUCTURE AND COORDINATES OF ELASTIC CENTER
4	APPLIED LOADS
5	STRUCTURE DEFLECTIONS
6	PILE DEFLECTIONS ALONG PILE AXIS
7	PILE FORCES ALONG PILE AXIS
8	PILE FORCES ALONG STRUCTURE AXIS

```
INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.

I>


 EXAMPLE PROBLEM NO 3
 HRENNIKOFF EXAMPLE


 NO. OF PILE ROWS =   5    B MATRIX IS CALCULATED FOR EACH ROW



**************************************************************************
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
  1    5  E =  .15E+07 PSI   IX =   322.06 IN**4   IY =   322.06 IN**4
          AREA =   63.6 IN**2   X =      9.00 IN      Y =      9.00 IN
          LENGTH =    30.0 FEET   ES =  31.230
          K1 =    .4107  K2 =   1.0000  K3 =   0.0000
          K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

```
ALLOWABLES:  COMPRESSIVE LOAD =     82.000 KIPS
             TENSILE LOAD =     40.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   5 IS

```
 .138E+04  0.         0.
0.          .265E+06  0.
0.         0.         0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-3.00	-5.000
2	-3.00	-2.500
3	-3.00	0.000
4	VERTICAL	3.000
5	VERTICAL	7.000

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.860E+05	-.237E+06	-.712E+07
-.237E+06	.125E+07	-.103E+08
-.712E+07	-.103E+08	.329E+10

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.664E-04	.142E-04	.188E-06
.142E-04	.386E-05	.429E-07
.188E-06	.429E-07	.847E-09

COORDINATES OF ELASTIC CENTER

EC1 = .003 EC2 = -.002

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-39.375	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.616E+00	-.332E-01	-.805E-03

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.610E+00	.117E+00	-.805E-03
2	-.602E+00	.140E+00	-.805E-03
3	-.595E+00	.163E+00	-.805E-03
4	-.616E+00	-.421E-02	-.805E-03
5	-.616E+00	.344E-01	-.805E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-.845	31.132	0.000	F
2	-.834	37.204	0.000	F
3	-.824	43.276	0.000	F
4	-.853	-1.117	0.000	
5	-.853	9.124	0.000	

TOTAL NO. FAILURES = 3 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-10.646	29.267	0.000
2	-12.556	35.031	0.000
3	-14.467	40.795	0.000
4	-.853	-1.117	0.000
5	-.853	9.124	0.000
SUM	-39.375	113.100	173.400

Pile No.	Computer Output F_1 (kips)	F_3 (kips)	Hrennikoff's Example F_1 (kips)	F_3 (kips)
1	0.845	31.132	0.84	31.2
2	0.834	37.204	0.83	37.2
3	0.824	43.276	0.82	43.2
4	0.853	-1.117	0.85	-1.0
5	0.853	9.124	0.85	9.1

Example Problem 4

Two-dimensional problem,
Hrennikoff's example
case 6a (medium soil)

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
$Mod_{sub} = 312.30$ psi	Pile resistance = 1.00
$I_x = 322.06$ in.^{4}	Participation factor for torsion = 0.0
$I_y = 322.06$ in.^{4}	
Area = 63.5 in.^{2}	Torsion modulus = 0.0
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Properties and loading condition for
example problem 4

```
2 HRENNIKOFF EXAMPLE
3 2
4 5 1 1
5 1 312.3
6 1 5 30.000 3
7 9.000
8 4
9 1500000.000
10 2
11 0. 1.0 0. 0.
12 1
13 82.000 40.000
14 2
15 1 3 -3.000
16 4 5 0.
17 -5.000 -2.5  0. 3.000 7.000
18 -39.375 113.1 173.4
E>FILE
STROM5 EDITED.
C>REPLACE,STROM5
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.


I>STROM5



 THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                CONTENTS
    1               PILE AND SOIL DATA
    2               PILE COORDINATES AND BATTER
    3               STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                    STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4               APPLIED LOADS
    5               STRUCTURE DEFLECTIONS
    6               PILE DEFLECTIONS ALONG PILE AXIS
    7               PILE FORCES ALONG PILE AXIS
    8               PILE FORCES ALONG STRUCTURE AXIS
```

```
INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.



I>
```

```
EXAMPLE PROBLEM NO 4
HRENNIKOFF EXAMPLE


NO. OF PILE ROWS =   5    B MATRIX IS CALCULATED FOR EACH ROW



**************************************************************************
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
 1   5  E =  .15E+07 PSI   IX =    322.06 IN**4   IY =   322.06 IN**4
        AREA =   63.6 IN**2   X =      9.00 IN      Y =      9.00 IN
        LENGTH =   30.0 FEET   ES = 312.300
        K1 =    .4107  K2 =   1.0000  K3 =   0.0000
        K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

```
ALLOWABLES:  COMPRESSIVE LOAD =     82.000 KIPS
             TENSILE LOAD =     40.000 KIPS
```

THE B MATRIX FOR PILES 1 THROUGH 5 IS

```
 .779E+04  0.        0.
0.         .265E+06  0.
0.        0.         0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-3.00	-5.000
2	-3.00	-2.500
3	-3.00	0.000
4	VERTICAL	3.000
5	VERTICAL	7.000

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.116E+06	-.232E+06	-.695E+07
-.232E+06	.125E+07	-.103E+08
-.695E+07	-.103E+08	.329E+10

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.205E-04	.427E-05	.566E-07
.427E-05	.171E-05	.144E-07
.566E-07	.144E-07	.468E-09

COORDINATES OF ELASTIC CENTER
EC1 = .003 EC2 = -.002

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-39.375	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.207E+00	.553E-01	.368E-03

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.172E+00	.139E+00	.368E-03
2	-.175E+00	.128E+00	.368E-03
3	-.179E+00	.118E+00	.368E-03
4	-.207E+00	.420E-01	.368E-03
5	-.207E+00	.243E-01	.368E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-1.338	36.790	0.000	
2	-1.365	34.014	0.000	
3	-1.392	31.237	0.000	
4	-1.611	11.137	0.000	
5	-1.611	6.454	0.000	

TOTAL NO. FAILURES = 0 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-12.903	34.479	0.000
2	-12.051	31.836	0.000
3	-11.199	29.194	0.000
4	-1.611	11.137	0.000
5	-1.611	6.454	0.000
SUM	-39.375	113.100	173.400

Results and calculations

Manual calculations for this example are presented in Hrennikoff's paper, case 6a. The computer results shown agree closely with the classical method results. For example, a comparison of the horizontal forces in each pile is shown below:

Pile No.	F_1 (kips) from Computer Output	F_1 (kips) from Hrennikoff's Example
1	1.338	1.34
2	1.365	1.37
3	1.392	1.39
4	1.611	1.61
5	1.611	1.61

The vertical pile forces also agree closely and are shown below:

Pile No.	F_3 (kips) from Computer Output	F_3 (kips) from Hrennikoff's Example
1	36.790	36.8
2	34.014	34.0
3	31.237	31.2
4	11.137	11.1
5	6.454	6.5

Example Problems 1-4

Pile Force ~ F3 (kips) Along Pile Axis
vs Subgrade Modulus (psi)

Pile No	Subgrade Modulus Value (psi)			
	0	3.123	31.23	312.3
1	23.589^{k}	27.395^{k}	31.132^{k}	36.790^{k}
2	41.394^{k}	39.282^{k}	37.204^{k}	34.014^{k}
3	59.199^{k}	51.170^{k}	43.276^{k}	31.237^{k}
4	-17.359^{k}	-9.167^{k}	-1.117^{k}	11.137^{k}
5	12.670^{k}	10.881^{k}	9.124^{k}	6.454^{k}

Pile Force - F3 (kips)

Example Problem 5

Two-dimensional Problem
Batter and vertical piles supporting a wall foundation with constant soil moduli

Properties:

$E = .15 \times 10^7$ psi (wood)

$Mod_{sub} = 312$

$I_x = 322$ in^4

$I_y = 322$ in^4

Area = 63.5 in^2

Length = 30 ft

Degree of fixity = 0.0

Pile resistance = 1.0

Participation factor for torsion = 0.0

Torsion modulus = 0.0

Loading:

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft
1	-40	0	113.1	0	173.4	0
2	-55	0	113.1	0	173.4	0

```
EXAMPLE PROB 5
RETAINING WALL
2
5 1 2
1 312
1 5 30 3
9
4
1500000
2
0 1 0 0
1
80 40
2
1 3 -2.4
4 5 0
-5 -2.5 0 3 7
-40 113.1 173.4
-55 113.1 173.4
```

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.	CONTENTS
1	PILE AND SOIL DATA
2	PILE COORDINATES AND BATTER
3	STIFFNESS AND FLEXIBILITY MATRICES FOR THE STRUCTURE AND COORDINATES OF ELASTIC CENTER
4	APPLIED LOADS
5	STRUCTURE DEFLECTIONS
6	PILE DEFLECTIONS ALONG PILE AXIS
7	PILE FORCES ALONG PILE AXIS
8	PILE FORCES ALONG STRUCTURE AXIS

INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8

INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.

I>

INPUT A FILE NAME FOR OUTPUT IN 7 CHARACTERS OR LESS.
HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.

I>

EXAMPLE PROB 5
RETAINING WALL

NO. OF PILE ROWS = 5 B MATRIX IS CALCULATED FOR EACH ROW

****

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
    1    5  E =  .15E+07 PSI    IX =   322.06 IN**4    IY =   322.06 IN**4
            AREA =    63.6 IN**2   X =      9.00 IN      Y =     9.00 IN
            LENGTH =    30.0 FEET    ES =  312.000
            K1 =     .4107  K2 =   1.0000  K3 =    0.0000
            K4 =    0.0020  K5 =   0.0000  K6 =    0.0000
```

```
        ALLOWABLES:  COMPRESSIVE LOAD =      80.000 KIPS
                     TENSILE LOAD =     40.000 KIPS
```

THE B-MATRIX FOR PILES 1 THROUGH 5 IS

```
  .778E+04  0.          0.
 0.          .265E+06  0.
 0.         0.          0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-2.40	-5.000
2	-2.40	-2.500
3	-2.40	0.000
4	VERTICAL	3.000
5	VERTICAL	7.000

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

```
  .153E+06  -.274E+06  -.822E+07
 -.274E+06   .121E+07  -.114E+08
 -.822E+07  -.114E+08   .324E+12
```

.186E-04	.480E-05	.641E-07
.480E-05	.210E-05	.196E-07
.641E-07	.196E-07	.541E-09

COORDINATES OF ELASTIC CENTER
EC1 = .004 EC2 = -.003

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
40.000	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.662E-01	.857E-01	.776E-03

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.102E-01	.148E+00	.776E-03
2	-.192E-01	.126E+00	.776E-03
3	-.292E-01	.105E+00	.776E-03
4	-.662E-01	.577E-01	.776E-03
5	-.662E-01	.205E-01	.776E-03

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-.080	39.113	0.000	
2	-.149	33.415	0.000	
3	-.219	27.717	0.000	
4	-.515	15.307	0.000	
5	-.515	5.431	0.000	

TOTAL NO. FAILURES = 0   LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-15.117	36.074	0.000
2	-12.990	30.787	0.000
3	-10.863	25.501	0.000
4	-.515	15.307	0.000
5	-.515	5.431	0.000
SUM	-40.000	113.100	173.400

****

******** LOADING CONDITION 2 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-55.000	113.100	173.400

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.345E+00	.136E-01	-.185E-03

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.317E+00	.135E+00	-.185E-03
2	-.315E+00	.140E+00	-.185E-03
3	-.313E+00	.145E+00	-.185E-03
4	-.345E+00	.203E-01	-.185E-03
5	-.345E+00	.292E-01	-.185E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-2.469	35.770	0.000	
2	-2.453	37.129	0.000	
3	-2.436	38.487	0.000	
4	-2.683	5.378	0.000	
5	-2.683	7.733	0.000	

TOTAL NO. FAILURES = 0 LOAD CASE 2

```
******************************************************************************

---------------------------------------------------------------------------

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)
   PILE     F1          F3          F5
-------------------------------------------------
   1    -16.037     32.069      0.000
   2    -16.544     33.329      0.000
   3    -17.052     34.590      0.000
   4     -2.683      5.378      0.000
   5     -2.683      7.733      0.000
-------------------------------------------------

   SUM  -55.000    113.100    173.400
-------------------------------------------------

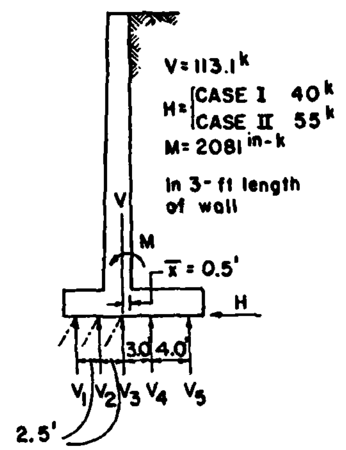
******************************************************************************

EXIT.
C>BYE
JOB PROCESSING CCUS     17.910
BYE 80/01/25. 14.46.24.

SELECT DESIRED SERVICE:
```

Computations

COMMON ANALYTICAL METHOD



LONGITUDINAL PILE SPACING

3'-0" C/C

$$\bar{x} = \frac{(3.0)+(3.0+4.0)-2.5-(2.5+2.5)}{5} = .5'$$

$$\Sigma d^2 = 5.5^2 + 3.0^2 + 2.5^2 + 6.5^2$$
$$= 87.75 \text{ ft}^2$$

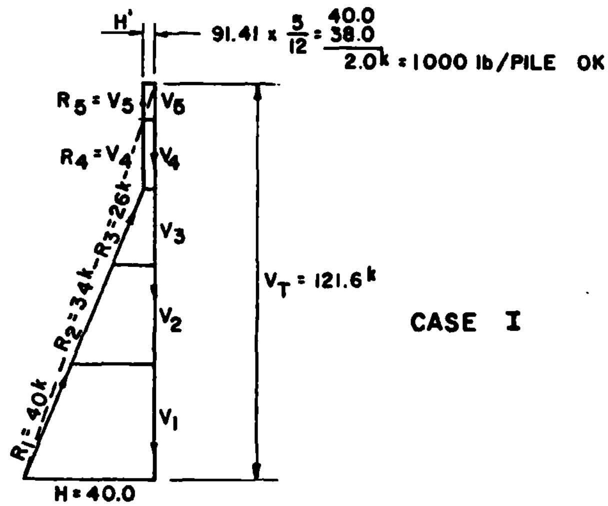
$$M' = 2081 + 113.1 \times 6'' = 2759.6''^k = 230'^k$$

$$V = \frac{113.1}{5} \pm \frac{230}{87.75} d$$
$$= 22.62^k \pm 2.62 d$$

$d_1 = 5.5$ $d_2 = 3.0$ $d_3 = .5$ $d_4 = -2.5$ $d_5 = -6.5$

$V_1 = 37.02^k$ $V_2 = 30.48^k$ $V_3 = 23.91^k$ $V_4 = 16.07^k$ $V_5 = 5.57^k$

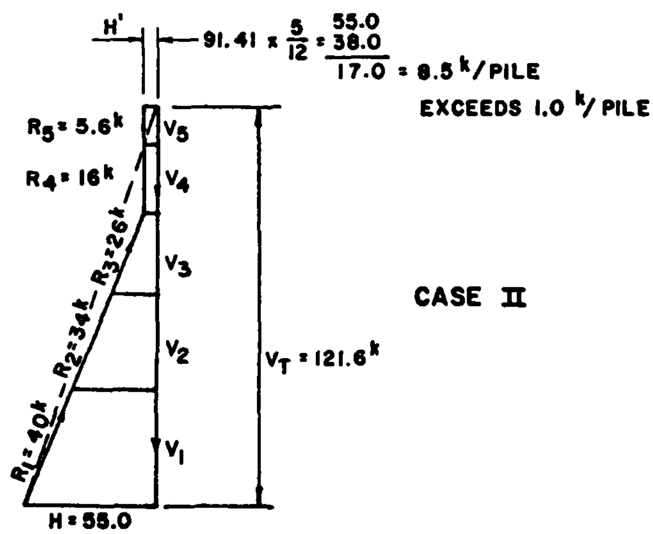
$V_{TOTAL} = 113.1^k$.



CASE I

$R_1 = 40^k$
$R_2 = 34^k$
$R_3 = 26^k$
$R_4 = 16^k$
$R_5 = 5.6^k$

SCALE : 1" = 100



CASE II

$R_1 = 40^k$
$R_2 = 34^k$
$R_3 = 26^k$
$R_4 = 16^k$
$R_5 = 5.6^k$

SCALE : 1" = 100

Comparison of Results - Pile Loads

CASE I

PILE NO.	COMPUTER OUTPUT KIPS	ANALYTICAL METHOD KIPS	COMMENTS
1	39.1	40	
2	33.4	34	
3	27.7	26	Use 20T piles
4	15.3	16	
5	5.4	5.6	

CASE II

PILE NO.	COMPUTER OUTPUT KIPS	ANALYTICAL METHOD KIPS	COMMENTS
1	35.8	40	Reference Foundation Design
2	37.1	34	by Wayne Teng. Using the
3	38.5	26	analytical method Piles 4 &
4	5.4	16	5 have horizontal load
5	7.7	5.6	$>1.0^k$: The Pile batter
			for Piles 1, 2, 3 must be
			increased.

EXAMPLE PROBLEMS 6 AND 7

Two-dimensional Problems

Retaining Wall on Piles from:
"Substructure Analysis and Design"
by Dr. Paul Andersen

Comparison of Computer Output
with Elastic Center Method and
a Common Analytical Method

Example Problems 6 and 7 illustrate how the computer can be used to analyze pile forces for a retaining wall founded on piles. The computer results are compared with the results obtained by hand computation methods commonly used by civil engineers. The physical pile layout for examples 6 and 7 is shown

Example Problem no. 6 compares the results obtained by the computer method with those obtained by the elastic center method assuming the soil offers no lateral support; in other words, the subgrade modulus is zero. The computer results agree closely with the elastic center method results. A description of the elastic center method can be found in "Substructure Analysis and Design" by Dr. Paul Andersen. This method, however, is limited to pile groups consisting of hinged piles arranged in two subgroups whose centerlines intersect.

Example Problem no. 7 compares the results obtained by the computer method with the results obtained by a common analytical method. A description of the common analytical method can be found in "Foundation Engineering" by Ralph Peck, Walter E. Hanson, and Thomas H. Thornburn, and in "Foundation Design" by Wayne Teng. Example Problem no. 7 demonstrates that the individual pile forces

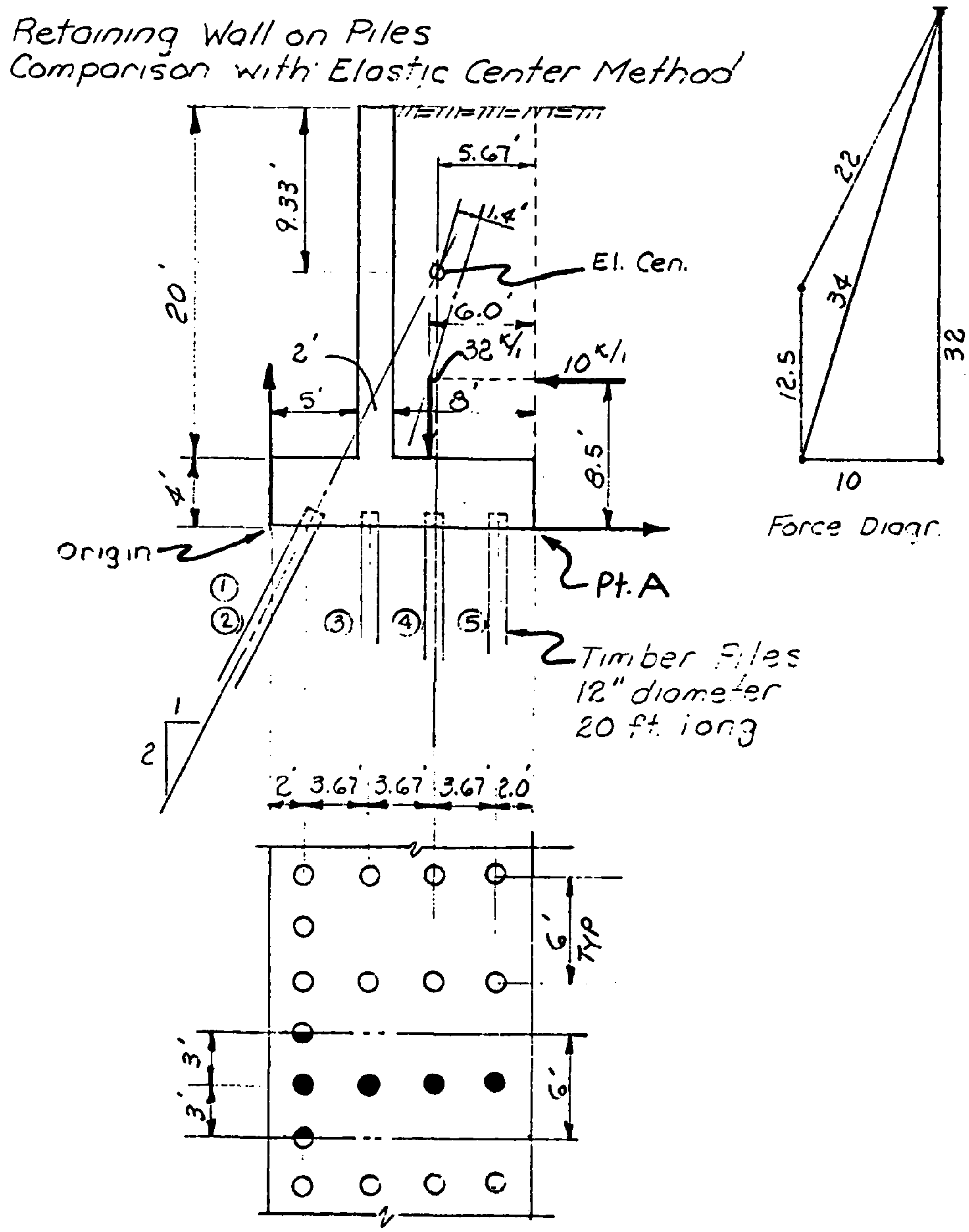


Figure E2. Physical problem for example problems 6 and 7

obtained by this method are not always correct. For this particular problem, the results do not compare favorably with the results obtained by the computer method. A subgrade modulus of 312 psi was used for the computer method.

Example Problem 6

Two-dimensional problem
retaining wall on piles
no lateral soil support

Properties	
$E = 0.18 \times 10^7$ psi	Degree of fixity = 0.0
Mod_{sub} = 10 psi 0	Pile resistance = 1.0
$I_x = 1017.87$ in.^{4}	Participation factor for torsion = 0.0
$I_y = 1017.87$ in.^{4}	Torsion modulus = 0.0
Area = 113.1 in.^{2}	
Length = 20.0 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-60.0	192.0	-1218.0

Properties and loading conditions
for example problem 6

```
(CR)

welcome to the bcs network
your access port is swy 78

select desired service: EKS1 (CR)

 79/08/24. 12.44.41.
EKS1 1751.N0460.429  79/08/19.DS-0   02.24.12. 79/08/21.
USER NUMBER: [redacted] (CR)
[redacted]
PASSWORD: [redacted] (CR)
[redacted]

TERMINAL -  205, TTY
RECOVER/USER ID: CHAMBERS (CR)

 TO COMMENT ON EKS SERVICE QUALITY, SEE MSG,26.
 THE IMPLEMENTATION DATE HAS BEEN REVISED FOR LATER IN SEPTEMBER.
 USER NOTIFICATION WILL BE TWO WEEKS IN ADVANCE OF IMPLEMENTATION.
 CONSOLIDATED PLOT LIBRARY IMPLEMENTATION WILL BE 08/26/79 FOR EKS1
 AND EKS2, 09/09/79 ON EKS3.
 FOR LABOR DAY HOLIDAY SCHEDULE, PLEASE SEE MSG,4.

 12.45.37.CORPS 'HOTNEWS/UN=CECELB' UPDATED 2 AUG79
 12.45.37.SUBJECT IS CERL FILES
 12.45.39.INVENTORY OF DAMS UPDATED 8 AUG 79 SEE HOTDAM/UN=CEC1AT
$
C>CMEDIT,STROM1 (CR)       create file STROM1
NEW FILE.
I>1 SAMPLE PROBLEM NO. 1        input line numbers and data (CR)
I>2 RETAINING WALL ON PILES
I>3 2
I>4 5 1 1
I>6 1 10
I>7 1 5 20 3
I>8 12
I>9 2
I>10 2
I>11 0 1 0 0
I>12 1
I>13 80 40
I>14 2
I>15 1 2 -2
I>16 3 5 0
I>17 2 2 5.67 9.34 13
I>18 -60 192.-1218
I> (CR)  END OF DATA INPUT
E>TOP -  GO TO TOP OF FILE
E>P 18   print 18 lines
```

```
1 SAMPLE PROBLEM NO. 6
2 RETAINING WALL ON PILES
3 2
4 5 1 1
6 1 10
7 1 5 20 3
8 12
9 2
10 2
11 0 1 0 0
12 1
13 80 40
14  2
15 1 2 -2
16  3 5 0
17 2 2 5.67 2.34 13
18 -60 192 -1218
D>FILE      EXIT EDIT MODE
STROM1 EDITED.
C>REPLACE,STROM1    SAVE UNDER STROM1
C>OLD,CORPS/UN=CECELB  } execute program
C>CALL,CORPS,X0034

 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.

I>STROM1 (CR)

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.          CONTENTS
    1              PILE AND SOIL DATA
    2              PILE COORDINATES AND BATTER
    3              STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                   STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4              APPLIED LOADS
    5              STRUCTURE DEFLECTIONS
    6              PILE DEFLECTIONS ALONG PILE AXIS
    7              PILE FORCES ALONG PILE AXIS
    8              PILE FORCES ALONG STRUCTURE AXIS

INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8 (CR)

INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.
 (CR)
```

```
INPUT A FILE NAME FOR OUTPUT IN 7 CHARACTERS OR LESS.
HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.

I> CR

SAMPLE PROBLEM NO. 6
RETAINING WALL ON PILES

NO. OF PILE ROWS =   5   B-MATRIX IS CALCULATED FOR EACH ROW

******************************************************************

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

  1   5  E =  .18E+07 PSI   IX =  1017.87 IN**4   IY =  1017.87 IN**4
         AREA =  113.1 IN**2   X =    12.00 IN      Y =    12.00 IN
         LENGTH =   20.0 FEET   ES =  10.000
         K1 =    .4107  K2 =   1.0000  K3 =   0.0000
         K4 =   0.0000  K5 =   0.0000  K6 =   0.0000

LENGTH OF PILES ( 20.00 FEET) IS INSUFFICIENT
FOR PILE GROUP -   1 MINIMUM ACCEPTABLE LENGTH IS   38.56 FEET
FOR SEMI-INFINITE BEAM ON ELASTIC FOUNDATION

         ALLOWABLES: COMPRESSIVE LOAD =    80.000 KIPS
                     TENSILE LOAD =     40.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH   5 IS

 .818E+03  0.        0.
 0.        .829E+06  0.
 0.        0.        0.

******************************************************************
```

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-2.00	2.000
2	-2.00	2.000
3	VERTICAL	5.670
4	VERTICAL	9.340
5	VERTICAL	13.000

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.336E+06	-.663E+06	.159E+08
-.663E+06	.382E+07	-.311E+09
.159E+08	-.311E+09	.352E+11

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.135E-04	.654E-05	.516E-07
.654E-05	.411E-05	.333E-07
.516E-07	.333E-07	.299E-09

COORDINATES OF ELASTIC CENTER
EC1 = .009 EC2 = .000

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-60.000	192.000	-1218.000

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.306E+00	-.901E-01	-.107E-02

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.303E+00	.793E-01	-.107E-02
2	-.303E+00	.793E-01	-.107E-02
3	-.306E+00	-.172E-01	-.107E-02
4	-.306E+00	.300E-01	-.107E-02
5	-.306E+00	.771E-01	-.107E-02

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU-CO-TE
1	-.247	65.747	0.000	F
2	-.247	65.747	0.000	F
3	-.250	-14.250	0.000	
4	-.250	24.905	0.000	F
5	-.250	63.954	0.000	

TOTAL NO. FAILURES = 3 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-29.624	58.696	0.000
2	-29.624	58.696	0.000
3	-.250	-14.250	0.000
4	-.250	24.905	0.000
5	-.250	63.954	0.000
SUM	-60.000	192.000	-1218.000

$$$

EXIT.
C>BYE LOGOFF
JOB PROCESSING CCUS 19.204
BYE 79/08/24. 12.55.43.

select desired service:

By elastic center method

Consider piles 1 & 2 are pile group A
and piles 3, 4 & 5 are pile group B

Then by the elastic center method

$$P_A = \frac{R_A}{n_A} + \frac{r}{\Sigma r^2} M \quad \text{and} \quad P_B = \frac{R_B}{n_B} + \frac{r}{\Sigma r^2} M$$

Where: R_A & R_B are force components in directions of A & B pile groups

n_A & n_B are the number of piles in A & B pile groups

r is the distance from the elastic center to the pile.

M is the moment of the applied loads about the elastic center

$$\Sigma r^2 = 2(3.67)^2 = 26.94$$

$$\text{Pile 1} = 22.0(3) = \text{.........} \; 66.00$$

$$\text{Pile 2} = 22.0(3) = \text{.........} \; 66.00$$

$$\text{Pile 3} = 6\left[\frac{12.5}{3} - \frac{34(1.4)3.67}{26.94}\right] = -13.91$$

$$\text{Pile 4} = 6\left[\frac{12.5}{3}\right] = \text{.........} \; 25.00$$

$$\text{Pile 5} = 6\left[\frac{12.5}{3} + \frac{34(1.4)3.67}{26.94}\right] = 63.91$$

Comparison of results obtained by the computer method with the results obtained by the elastic center method.

Pile Forces along Pile Axis

Pile No.	Computer Output F_3 (Kips)	Elastic Center F_3 (Kips)
1	65.747	66.00
2	65.747	66.00
3	-14.250	-13.91
4	24.905	25.00
5	63.954	63.91

Example Problem 7

Two-dimensional problem
retaining wall on piles
subgrade modulus = 312.0 psi
(medium soil)

Properties	
$E = 0.18 \times 10^7$ psi	Degree of fixity = 0.0
Mod_{sub} = 312.0 psi	Pile resistance = 1.0
I_x = 1017.87 in.^{4}	Participation factor for torsion = 0.0
I_y = 1017.87 in.^{4}	Torsion modulus = 0.0
Area = 113.1 in.^{2}	
Length = 20.0 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
L	-60.0	192.0	1218.0

Properties and loading conditions
for example problem 7

```
10 EXAMPLE PROBLEM NO 7
20 RETAINING WALL ON PILES
30 2
40 5 1 1
50 1 312
60 1 5 20 3
70 12
80 2
90 2
100 0 1 0 0
110 1
120 80 40
130 2
140 1 2 -2
150 3 5 0
160 2 2 5.67 9.34 13
170 -60 192 -1218
EOI.

 07.02.03. WARNING


SYSTEM DOWN IN 5 MIN...ECS ERRORS
E>FILE
STROM1 EDITED.
C>REPLACE,STROM1
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.


I>STROM1
```

```
THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                CONTENTS
    1              PILE AND SOIL DATA
    2              PILE COORDINATES AND BATTER
    3              STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                   STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4              APPLIED LOADS
    5              STRUCTURE DEFLECTIONS
    6              PILE DEFLECTIONS ALONG PILE AXIS
    7              PILE FORCES ALONG PILE AXIS
    8              PILE FORCES ALONG STRUCTURE AXIS


INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN



I>


EXAMPLE PROBLEM NO 7
RETAINING WALL ON PILES


NO. OF PILE ROWS =   5    B MATRIX IS CALCULATED FOR EACH ROW



***************************************************************************
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
   1   5  E =  .18E+07 PSI   IX =  1017.87 IN**4   IY =  1017.87 IN**4
          AREA =  113.1 IN**2   X =     12.00 IN     Y =    12.00 IN
          LENGTH =   20.0 FEET   ES = 312.000
          K1 =    .4107  K2 =   1.0000  K3 =   0.0000
          K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

```
         ALLOWABLES:  COMPRESSIVE LOAD =     80.000 KIPS
                      TENSILE LOAD =     40.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   5 IS

```
  .108E+05  0.         0.
 0.          .829E+06  0.
 0.         0.         0.
```

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW	BATTER	U1 (FT)
1	-2.00	2.000
2	-2.00	2.000
3	VERTICAL	5.670
4	VERTICAL	9.340
5	VERTICAL	13.000

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.381E+06	-.655E+06	.157E+08
-.655E+06	.382E+07	-.311E+09
.157E+08	-.311E+09	.352E+11

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.787E-05	.377E-05	.298E-07
.377E-05	.273E-05	.224E-07
.298E-07	.224E-07	.213E-09

COORDINATES OF ELASTIC CENTER
EC1 = .009 EC2 = .000

######## LOADING CONDITION 1 ########

##

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-60.000	192.000	-1218.000

##

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-.183E+00	-.293E-01	-.592E-03

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-.171E+00	.684E-01	-.592E-03
2	-.171E+00	.684E-01	-.592E-03
3	-.183E+00	.110E-01	-.592E-03
4	-.183E+00	.370E-01	-.592E-03
5	-.183E+00	.630E-01	-.592E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CO TE
1	-1.842	56.762	0.000	
2	-1.842	56.762	0.000	
3	-1.978	9.108	0.000	
4	-1.978	30.723	0.000	
5	-1.978	52.279	0.000	

TOTAL NO. FAILURES = 0 LOAD CASE 1

****

B. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-27.032	49.945	0.000
2	-27.032	49.945	0.000
3	-1.978	9.108	0.000
4	-1.978	30.723	0.000
5	-1.978	52.279	0.000
SUM	-60.000	192.000	-1218.000

By Common Analytical Method

Forces

$\Sigma H = 10\ ^{K}/_{FT}$

$\Sigma V = 32\ ^{K}/_{FT}$

Moments about A

$\Sigma M_A = 10(8.5) + 32(6.0) = 277\ ^{1K}/_{1}$

Point A to Resultant

$\frac{277}{32} = 8.66'$

CG of Piles and Eccentricity

$\frac{2(11.0) + 1(7.34) + 1(3.67)}{5} = 6.60$ Ft. left of row 5

$e = 8.66 - 8.60 = .06'$

Σd^2 and Section Moduli:

$\Sigma d^2 = 2(4.41)^2 + (.74)^2 + (2.93)^2 + (6.60)^2 = 91.44$ pile-ft^2

Rows 1 & 2	$91.44 \div 4.41 = 20.74$ pile-ft
Row 3	$91.44 \div .74 = 123.57$ pile-ft
Row 4	$91.44 \div 2.93 = 31.21$ pile-ft
Row 5	$91.44 \div 6.60 = 13.85$ pile-ft

Loads and Moments on 6 Ft Strip

$\Sigma H = 10(6) = 60^{K}$

$\Sigma V = 32(6) = 192^{K}$

$\Sigma M = \Sigma Ve = 192(.06) = 11.52^{\prime K}$

Vertical Components of Pile Reactions

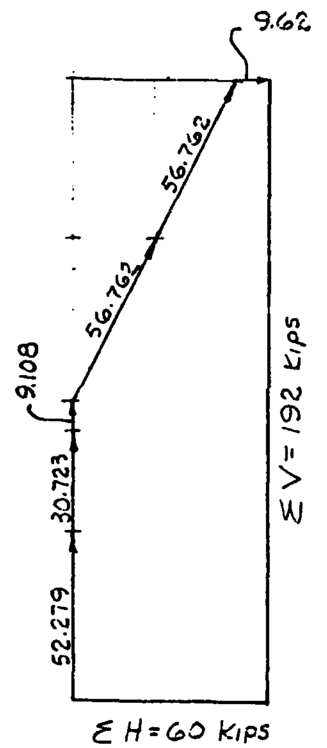
Rows 1 & 2 $\quad \frac{192}{5} + \frac{11.52}{20.74} = 38.96^{k}$

Row 3 $\quad \frac{192}{5} + \frac{11.52}{123.57} = 38.49^{k}$

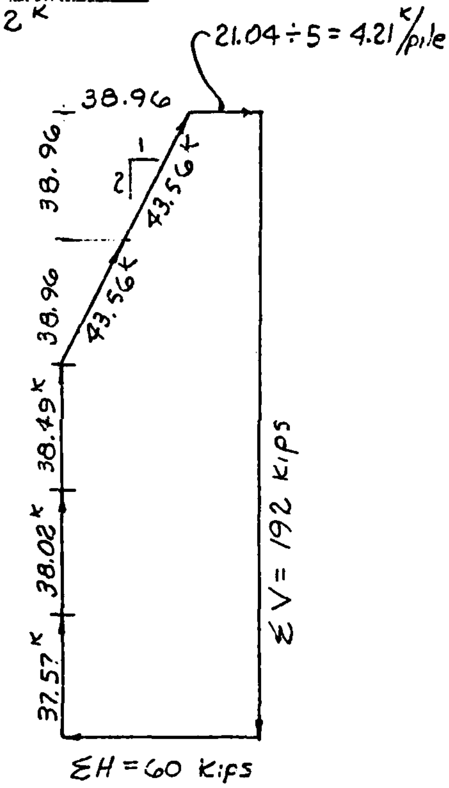
Row 4 $\quad \frac{192}{5} - \frac{11.52}{31.21} = 38.02^{k}$

Row 5 $\quad \frac{192}{5} - \frac{11.52}{13.85} = 37.57^{k}$

$\qquad\qquad\qquad\qquad 192^{k}$



Pile Force Diagram
Computer Method



Pile Force Diagram
Common Analytical Method

Pile Forces Along Pile Axis

Pile No.	Computer Output F3 (kips)	Comm. Anal Method F3 (kips)
1	56.762	43.56
2	56.762	43.56
3	9.108	38.49
4	30.723	38.02
5	52.279	37.57

Comparison of the results obtained by the Computer Method with the results obtained by the Common Analytical Method.

EXAMPLE PROBLEMS 8 AND 9

3-dimensional Problems
Aschenbrenner's Example
Comparison of Computer Output
with Aschenbrenner's Calculations

Example problems 8 and 9 illustrate how the computer can be used to analyze 3-dimensional pile foundations problems. The following examples were taken from Aschenbrenner's paper, entitled "Three-Dimensional Analysis of Pile Foundations", published in the ASCE Journal of the Structural Division, Vol. 93, paper no. 5097, Feb. 1967, pp. 201-219. The computer results are compared with Aschenbrenner's calculations. The physical pile layout for example problems 8 and 9 is shown in Figure E3.

Example Problem no. 8 compares the results of the computer method with Aschenbrenner's calculations for a subgrade modulus of 35 pci, assuming the subgrade modulus varies linearly with depth.

Example Problem no. 9 compares the results of the computer method with Aschenbrenner's calculations for a subgrade modulus equal to zero; in other words, assuming the soil offers no lateral support.

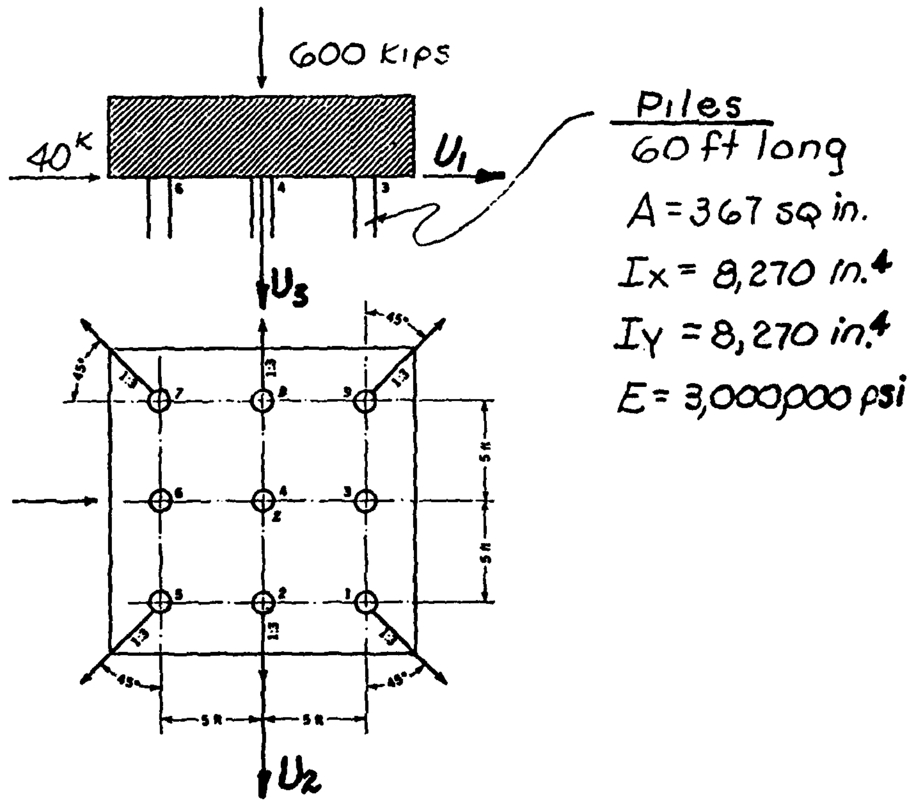


Figure E3. Physical problem for example problems 8 and 9

Example Problem 8

Three-dimensional problem
Aschenbrenner example
subgrade modulus = 35 pci

Properties	
E = 3,000,000 psi	Degree of fixity = 0.0
Mod_{sub} = 35.0 pci	Pile resistance = 1.0
I_x = 8,270 $in.^4$	Participation factor for torsion = 0.0
I_y = 8,270 $in.^4$	Torsion modulus = 0.0
Area = 367.0 $in.^2$	
Length = 60.0 ft	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	40.0	0.0	600.0	0.0	0.0	0.0

Properties and loading conditions
for example problem 8

```
10 EXAMPLE PROBLEM NO 8, ES=35
20 ASCHENBRENNER CHECK
30 3
40 9 1 1
50 2 35
60 1 9 60. 2
70 8270. 8270. 367. 16. 16.
80 4
90 3000000.
100 2
110 0. 1. 0. 0.
120 10. 10. 10. 150. 10. 10. 150. 10.
130 0
140 3. 45. 5. 5. 0.
150 3. 90. 0. 5. 0.
160 0. 0. 5. 0. 0.
170 0. 0. 0. 0. 0.
180 3. 135. -5. 5. 0.
190 0. 0. -5. 0. 0.
200 3. 225. -5. -5. 0.
210 3. 270. 0. -5. 0.
220 3. 315. 5. -5. 0.
230 40. 0. 600. 0. 0. 0.
EOI.
E>FILE
STROM3 EDITED.




C>REPLACE,STROM3
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034


 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.


I>STROM3
```

```
THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.              CONTENTS
    1             PILE AND SOIL DATA
    2             PILE COORDINATES AND BATTER
    3             STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                  STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4             APPLIED LOADS
    5             STRUCTURE DEFLECTIONS
    6             PILE DEFLECTIONS ALONG PILE AXIS
    7             PILE FORCES ALONG PILE AXIS
    8             PILE FORCES ALONG STRUCTURE AXIS


INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.



I>


 INPUT A FILE NAME FOR OUTPUT IN 7 CHARACTERS OR LESS.
 HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.



I>


 EXAMPLE PROBLEM NO 8, ES=35
 ASCHENBRENNER CHECK


 NO. OF PILES =   9    B MATRIX IS CALCULATED FOR EACH PILE
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
   1   9  E =  .30E+07 PSI   IX =  8270.00 IN**4   IY =  8270.00 IN**4
          AREA =  367.0 IN**2   X =     16.00 IN     Y =     16.00 IN
          LENGTH =    60.0 FEET   ES =  35.000
          K1 =     .4107  K2 =   1.0000  K3 =   0.0000
          K4 =    0.0000  K5 =   0.0000  K6 =   0.0000
```

```
ALLOWABLES:  COMBINED BENDING FOR TENSION =     10.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =     10.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =     10.000 KIP-FT
             COMBINED BENDING FOR COMPRESSION =   150.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =     10.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =     10.000 KIP-FT
             COMPRESSIVE LOAD =   150.000 KIPS
             TENSILE LOAD =    10.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   9 IS

.499E+05	0.	0.	0.	0.	0.
0.	.499E+05	0.	0.	0.	0.
0.	0.	.153E+07	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.	BATTER	ANGLE	U1(FT)	U2(FT)	U3(FT)
1	3.00	45.	5.000	5.000	0.000
2	3.00	90.	0.000	5.000	0.000
3	VERTICAL	0.	5.000	0.000	0.000
4	VERTICAL	0.	0.000	0.000	0.000
5	3.00	135.	-5.000	5.000	0.000
6	VERTICAL	0.	-5.000	0.000	0.000
7	3.00	225.	-5.000	-5.000	0.000
8	3.00	270.	0.000	-5.000	0.000
9	3.00	315.	5.000	-5.000	0.000

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.745E+06	-.212E-03	.769E-03	-.922E-01	-.753E+08	-.894E-07
-.212E-03	.104E+07	0.	.129E+09	-.540E-01	-.191E-01
.769E-03	0.	.129E+08	-.477E-06	.477E-06	.146E+00
-.922E-01	.129E+09	-.477E-06	.298E+11	0.	-.439E+01
-.753E+08	-.540E-01	.477E-06	0.	.309E+11	.387E-04
-.894E-07	-.191E-01	.146E+00	-.439E+01	.387E-04	.215E+10

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.178E-05	-.196E-15	-.106E-15	.635E-17	.434E-08	-.409E-23
-.196E-15	.206E-05	-.328E-21	-.886E-08	.312E-17	.184E-18
-.106E-15	-.328E-21	.777E-07	.265E-23	-.259E-18	-.527E-17
.635E-17	-.886E-08	.265E-23	.717E-10	-.970E-26	.675E-19
.434E-08	.312E-17	-.259E-18	-.179E-25	.430E-10	-.592E-24
-.409E-23	.184E-18	-.527E-17	.675E-19	-.592E-24	.464E-09

******** LOADING CONDITION 1 ********

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
40.000	0.000	600.000	0.000	0.000	0.000

##

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
.713E-01	-.784E-11	.466E-01	.254E-12	.174E-03	-.316E-11

##

PILE	X1	X2	X3	X4	X5	X6
1	.364E-01	-.504E-01	.503E-01	.117E-03	.123E-03	.389E-04
2	-.147E-01	-.713E-01	.442E-01	.165E-03	-.316E-12	.549E-04
3	.713E-01	-.198E-09	.362E-01	.254E-12	.174E-03	-.316E-11
4	.713E-01	-.784E-11	.466E-01	.254E-12	.174E-03	-.316E-11
5	-.659E-01	-.504E-01	.382E-01	.117E-03	-.123E-03	.389E-04
6	.713E-01	.182E-09	.570E-01	.254E-12	.174E-03	-.316E-11
7	-.659E-01	.504E-01	.382E-01	-.117E-03	-.123E-03	-.389E-04
8	-.147E-01	.713E-01	.442E-01	-.165E-03	.441E-12	-.549E-04
9	.364E-01	.504E-01	.503E-01	-.117E-03	.123E-03	-.389E-04

##

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB BU CO TE
1	1.814	-2.514	76.858	0.000	0.000	0.000	.51	
2	-.735	-3.555	67.606	0.000	0.000	0.000	.45	
3	3.555	-.000	55.321	0.000	0.000	0.000	.37	
4	3.555	-.000	71.263	0.000	0.000	0.000	.48	
5	-3.284	-2.514	58.354	0.000	0.000	0.000	.39	
6	3.555	.000	87.205	0.000	0.000	0.000	.58	
7	-3.284	2.514	58.354	0.000	0.000	0.000	.39	
8	-.735	3.555	67.606	0.000	0.000	0.000	.45	
9	1.814	2.514	76.858	0.000	0.000	0.000	.51	

TOTAL NO. FAILURES = 0 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	20.180	16.625	72.340	0.000	0.000	0.000
2	3.555	20.682	64.369	0.000	0.000	0.000
3	3.555	-.000	55.321	0.000	0.000	0.000
4	3.555	-.000	71.263	0.000	0.000	0.000
5	-9.068	12.623	56.398	0.000	0.000	0.000
6	3.555	.000	87.205	0.000	0.000	0.000
7	-9.068	-12.623	56.398	0.000	0.000	0.000
8	3.555	-20.682	64.369	0.000	0.000	0.000
9	20.180	-16.625	72.340	0.000	0.000	0.000
SUM	40.000	-.000	600.000	0.000	-.000	.000

Pile Forces Along Pile Axis

Pile No.	Computer Output F3 (kips)	Aschen-brenner F3 (kips)
1	76.858	79.7
2	67.606	67.5
3	55.321	49.6
4	71.263	71.3
5	58.354	55.3
6	87.205	93.0
7	58.354	55.3
8	67.606	67.5
9	76.858	79.7

Comparison of the results obtained by the Computer Method with the results obtained by Aschenbrenner.

Example Problem 9

Three-dimensional problem
Aschenbrenner example
no lateral soil pressure

Properties	
E = 3,000,000 psi	Degree of fixity = 0.0
Mod_{sub} = .1pci ≈ 0	Pile resistance = 1.0
I_x = 8,270 in.^{4}	Participation factor for torsion = 0.0
I_y = 8,270 in.^{4}	Torsion modulus = 0.0
Area = 367.0 in.^{2}	
Length = 60.0 ft	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	40.0	0.0	600.0	0.0	0.0	0.0

Properties and loading conditions
for example problem 9

```
10 EXAMPLE PROBLEM NO 9, ES=0
20 ASCHENBRENNER CHECK
30 3
40 9 1 1
50 2 .1
60 1 9 60. 2
70 8270. 8270. 367. 16. 16.
80 4
90 3000000.
100 2
110 0. 1. 0. 0.
120 10. 10. 10. 150. 10. 10. 150. 10.
130 0
140 3. 45. 5. 5. 0.
150 3. 90. 0. 5. 0.
160 0. 0. 5. 0. 0.
170 0. 0. 0. 0. 0.
180 3. 135. -5. 5. 0.
190 0. 0. -5. 0. 0.
200 3. 225. -5. -5. 0.
210 3. 270. 0. -5. 0.
220 3. 315. 5. -5. 0.
230 40. 0. 600. 0. 0. 0.
EOI.
```

```
C>REPLACE,STROM3
C>OLD,CORPS/UN=CECELB
C>CALL,CORPS,X0034
```

```
 INPUT DATA FILE NAME IN 7 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.
```

```
I>STROM3
```

```
THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                 CONTENTS
    1              PILE AND SOIL DATA
    2              PILE COORDINATES AND BATTER
    3              STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                   STRUCTURE AND COORDINATES OF ELASTIC CENTER
    4              APPLIED LOADS
    5              STRUCTURE DEFLECTIONS
    6              PILE DEFLECTIONS ALONG PILE AXIS
    7              PILE FORCES ALONG PILE AXIS
    8              PILE FORCES ALONG STRUCTURE AXIS


INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS.
I>1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 8 IN 7 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
 HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.



I>


 INPUT A FILE NAME FOR OUTPUT IN 7 CHARACTERS OR LESS.
 HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.



I>


 EXAMPLE PROBLEM NO 9, ES=0
 ASCHENBRENNER CHECK


 NO. OF PILES =    9    B MATRIX IS CALCULATED FOR EACH PILE
```

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

```
  1   9  E =  .30E+07 PSI   IX =  8270.00 IN**4   IY =  8270.00 IN**4
         AREA =  367.0 IN**2   X =    16.00 IN     Y =    16.00 IN
         LENGTH =   60.0 FEET   ES =     .100
         K1 =    .4107  K2 =   1.0000  K3 =   0.0000
         K4 =   0.0000  K5 =   0.0000  K6 =   0.0000
```

```
ALLOWABLES:  COMBINED BENDING FOR TENSION =     10.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =     10.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =     10.000 KIP-FT
             COMBINED BENDING FOR COMPRESSION =   150.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =    10.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =    10.000 KIP-FT
             COMPRESSIVE LOAD =   150.000 KIPS
             TENSILE LOAD =    10.000 KIPS
```

THE B MATRIX FOR PILES   1 THROUGH   9 IS

.148E+04	0.	0.	0.	0.	0.
0.	.148E+04	0.	0.	0.	0.
0.	0.	.153E+07	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.

****

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.	BATTER	ANGLE	U1(FT)	U2(FT)	U3(FT)
1	3.00	45.	5.000	5.000	0.000
2	3.00	90.	0.000	5.000	0.000
3	VERTICAL	0.	5.000	0.000	0.000
4	VERTICAL	0.	0.000	0.000	0.000
5	3.00	135.	-5.000	5.000	0.000
6	VERTICAL	0.	-5.000	0.000	0.000
7	3.00	225.	-5.000	-5.000	0.000
8	3.00	270.	0.000	-5.000	0.000
9	3.00	315.	5.000	-5.000	0.000

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

.319E+06	-.219E-03	.794E-03	-.952E-01	-.778E+08	-.154E-06
-.219E-03	.624E+06	-.373E-08	.133E+09	-.558E-01	-.197E-01
.794E-03	-.373E-08	.128E+08	0.	-.477E-06	.151E+00
-.952E-01	.133E+09	0.	.297E+11	0.	-.453E+01
-.778E+08	-.558E-01	-.477E-06	0.	.308E+11	.396E-04
-.154E-06	-.197E-01	.151E+00	-.453E+01	.396E-04	.641E+08

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

.815E-05	-.169E-13	-.504E-15	.102E-15	.206E-07	.685E-20
-.169E-13	.318E-04	.922E-20	-.142E-06	.148E-16	-.253E-15
-.504E-15	.922E-20	.778E-07	-.412E-22	-.127E-17	-.183E-15
.102E-15	-.142E-06	-.412E-22	.667E-09	-.345E-23	.351E-17
.206E-07	.148E-16	-.127E-17	-.324E-23	.843E-10	-.272E-23
.685E-20	-.253E-15	-.183E-15	.351E-17	-.272E-23	.156E-07

******** LOADING CONDITION 1 ********

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
40.000	0.000	600.000	0.000	0.000	0.000

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
.326E+00	-.678E-09	.467E-01	.407E-11	.823E-03	-.110E-09

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	.220E+00	-.231E+00	.704E-01	.552E-03	.582E-03	.184E-03
2	-.148E-01	-.326E+00	.443E-01	.780E-03	-.437E-11	.260E-03
3	.326E+00	-.728E-08	-.265E-02	.407E-11	.823E-03	-.110E-09
4	.326E+00	-.678E-09	.467E-01	.407E-11	.823E-03	-.110E-09
5	-.249E+00	-.231E+00	.182E-01	.552E-03	-.582E-03	.184E-03
6	.326E+00	.593E-08	.961E-01	.407E-11	.823E-03	-.110E-09
7	-.249E+00	.231E+00	.182E-01	-.552E-03	-.582E-03	-.184E-03
8	-.148E-01	.326E+00	.443E-01	-.780E-03	.496E-11	-.260E-03
9	.220E+00	.231E+00	.704E-01	-.552E-03	.582E-03	-.184E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB BU CO TE
1	.326	-.342	107.650	0.000	0.000	0.000	.72	
2	-.022	-.484	67.758	0.000	0.000	0.000	.45	
3	.484	-.000	-4.046	0.000	0.000	0.000	.40	
4	.484	-.000	71.424	0.000	0.000	0.000	.48	
5	-.370	-.342	27.866	0.000	0.000	0.000	.19	
6	.484	.000	146.893	0.000	0.000	0.000	.98	F
7	-.370	.342	27.866	0.000	0.000	0.000	.19	
8	-.022	.484	67.758	0.000	0.000	0.000	.45	
9	.326	.342	107.650	0.000	0.000	0.000	.72	

TOTAL NO. FAILURES = 1 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	24.532	24.048	102.023	0.000	0.000	0.000
2	.484	21.406	64.288	0.000	0.000	0.000
3	.484	-.000	-4.046	0.000	0.000	0.000
4	.484	-.000	71.424	0.000	0.000	0.000
5	-5.741	6.225	26.553	0.000	0.000	0.000
6	.484	.000	146.893	0.000	0.000	0.000
7	-5.741	-6.225	26.553	0.000	0.000	0.000
8	.484	-21.406	64.288	0.000	0.000	0.000
9	24.532	-24.048	102.023	0.000	0.000	0.000
SUM	40.000	.000	600.000	.000	-.000	.000

Pile Forces Along Pile Axis

Pile No.	Computer Output F3 (kips)	Aschen-brenner F3 (kips)
1	107.650	112.5
2	67.758	67.5
3	-4.046	-13.1
4	71.424	71.3
5	27.866	22.5
6	146.893	155.7
7	27.866	22.5
8	67.758	67.5
9	107.650	112.5

Comparison of the results obtained by the Computer Method with the results obtained by Aschenbrenner.

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

CASE Task Group on Pile Foundations.
Basic pile group behavior / by the CASE Task Group on Pile Foundations. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. ; available from National Technical Information Service, 1980.
22, [144] p. : ill. ; 27 cm. (Technical report - U. S. Army Engineer Waterways Experiment Station ; K-80-5)
Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C.
References: p. 21-22.

1. Computer programs. 2. Computerized simulation. 3. Design criteria. 4. Pile foundation design. 5. Pile foundations. I. United States. Army. Corps of Engineers. II. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Technical report ; K-80-5.
TA7.W34 no.K-80-5

WATERWAYS EXPERIMENT STATION REPORTS PUBLISHED UNDER THE COMPUTER-AIDED STRUCTURAL ENGINEERING (CASE) PROJECT

	Title	Date
Technical Report K-78-1	List of Computer Programs for Computer-Aided Structural Engineering	Feb 1978
Instruction Report O-79-2	User's Guide: Computer Program With Interactive Graphics for Analysis of Plane Frame Structures (CFRAME)	Mar 1979
Technical Report K-80-1	Survey of Bridge-Oriented Design Software	Jan 1980
Technical Report K-80-2	Evaluation of Computer Programs for the Design/Analysis of Highway and Railway Bridges	Jan 1980
Instruction Report K-80-1	User's Guide: Computer Program for Design/Review of Curvilinear Conduits/Culverts (CURCON)	Feb 1980
Instruction Report K-80-3	A Three-Dimensional Finite Element Data Edit Program	Mar 1980
Instruction Report K-80-4	A Three-Dimensional Stability Analysis/Design Program (3DSAD)	
	Report 1: General Geometry Module	Jun 1980
Technical Report K-80-4	Documentation of Finite Element Analyses	
	Report 1: Longview Outlet Works Conduit	Dec 1980
	Report 2: Anchored Wall Monolith, Bay Springs Lock	Dec 1980
Technical Report K-80-5	Basic Pile Group Behavior	Dec 1980